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1. $\alpha = \frac{g^2}{4\pi}$

$$\frac{dg_i}{d\ln\mu} = \frac{dg}{d\alpha}\frac{d\alpha}{d\ln\mu}$$

$$= -\frac{2\pi}{g}\left(\frac{\alpha^2}{2\pi}\right) Z_i$$

$$\left(\frac{d\alpha}{dg} = \frac{2\cancel{g}}{\cancel{4}\pi} = \frac{g}{2\pi}\right.$$

where $Z_i = \left[\frac{11}{3} C_{adj} - \frac{2}{3}\sum_i C_{f,i} - \frac{1}{6} C_{s,i}\right]_i$

$i = 1 \ldots 2 \ldots 3.$

$$\boxed{\frac{dg_i}{d\ln\mu} = \frac{-g_i^3}{16\pi^2} Z_i} \quad \ldots \quad (1)$$

We need to calculate Z_i for three sectors i.e SU(2), U(1) and SU(3). For brevity, we refer to Z for SU(3) as Z_s, and Z_2 for U(1) and Z_1 for SU(2).

Let's calculate for SU(2) sector

$$Z_1 = \frac{11}{3} C_{adj} - \frac{2}{3}\Sigma C_{f,i} - \frac{1}{6} C_{s,i}$$

$$= \left(\frac{11}{3}\right) 2 - \left(\frac{2}{3}\right)\Sigma C_{f,i} - \frac{1}{6} C_{s,i}$$

For SU(2), we have three fermion (L-handed) doublets (from quark sector) and one LH-fermion doublet from lepton sector giving total of 4.

They're in the fundamental representation of SU(2).

Also, We have one scalar boson (Higgs) doublet).

Hence,

$$Z_1 = \frac{22}{3} - \left(\frac{2}{3}\right)(4)\left(\frac{1}{2}\right) N_g - \frac{1}{6} N_h$$

Setting $N_g = 3$ and $N_h = 1$

$$= \frac{22}{3} - 4 - \frac{1}{6}$$

$$= \frac{44 - 24 - 1}{6}$$

$$= 19/6 \qquad \left(\frac{22}{3} - \frac{4F}{3} - \frac{1}{6}\right)$$

For SU(3):

$$Z_s = \frac{11}{3} C_{adj} - \frac{2}{3} \Sigma\, C_{f,i} - \underline{\underline{\frac{1}{6} C_{s,i}}}$$

We know that Higgs is a SU(3) singlet or color singlet and hence underlined term above vanishes (not contribute) ; $\boxed{\text{Sum runs over doublets}}$.

$$Z_s = \left(\frac{11}{3}\right) 3 - \frac{2}{3} \Sigma\, C_{f,i}$$

$$= \left(\frac{11}{3}\right) 3 - \left(\frac{2}{3}\right) 4 \left(\frac{1}{2}\right) N_g$$

$$= 11 - \frac{4}{3} N_g$$

$$= 7 \qquad (\text{for } N_g = 3)$$

Four triplets of SU(3) for each generation/family.

* Rule: Each quark-lepton family has 4 triplets of SU(3) and 4 doublets of SU(2)

Now, we calculate Z_2 for U(1) sector :-

Note that U(1) abelian has neutral gauge field i.e $\boxed{\delta A_\mu(x) = 0}$ ✓. and hence

$\frac{11}{3} C_{adj}$ term is 0 here.

$$Z_2 = \frac{-2}{3} \Sigma \, C_{f,i} - \frac{1}{6} C_{s,i}$$

Also the U(1) sector to be embedded into SU(5) Model should carry factor of 3/5 and the hyperchange square for each contribution.

$$Z_2 = -\frac{2}{3} \sum_{f,i} \frac{3}{5} Y_f^2 - \frac{1}{6} \sum_{s,i} \frac{3}{5} Y_s^2$$

In standard model, ~~[illegible] family~~ each family has 15 weyl fermions which are representations of $SU(3) \otimes SU(2) \otimes U(1)$.
Fermions can be shown to consist of following :

$$\left(3, 2, +\tfrac{1}{6}\right), \left(1, 2, -\tfrac{1}{2}\right), \left(3, 1, +\tfrac{2}{3}\right), \left(3, 1, -\tfrac{1}{3}\right),$$
$$\left(1, 1, -1\right)$$

where $(\cdot , \cdot , \cdot)$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ SU(3) & SU(2) & U(1)_Y \longrightarrow \text{hypercharge} \end{array}$$

$$Z_2 = \frac{-2}{5}\left[6\left(\frac{1}{6}\right)^2 + 2\left(\frac{-1}{2}\right)^2 + 3\left(\frac{2}{3}\right)^2 + 3\left(\frac{-1}{3}\right)^2 + (-1)^2\right] N_g$$

$$- \frac{1}{10}\, 2\left(\frac{1}{2}\right)^2 N_h$$

$$= \frac{-2}{5}\left[\frac{1}{6} + \frac{1}{2} + \frac{4}{3} + \frac{1}{3} + 1\right] N_g \quad - \frac{1}{20}$$

$$\boxed{N_h = 1}$$

Higgs form doublet of complex scalars $\left(1, 2, \frac{1}{2}\right)$

$$= -\frac{4}{3} N_g - \frac{1}{20}$$

for $N_g = 3$

$$Z_2 = \frac{-81}{20}$$

$$\begin{pmatrix} Z_{U(1)} \\ Z_{SU(2)} \\ Z_{SU(3)} \end{pmatrix} = \begin{pmatrix} Z_2 \\ Z_1 \\ Z_s \end{pmatrix} = \begin{pmatrix} -81/20 \\ 19/6 \\ 7 \end{pmatrix}$$

b) We know from previous assignment that α_i depends on energy scale μ as:

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_{cutoff})} - \frac{Z_i}{2\pi} \ln\left(\frac{M_{cut}}{M_Z}\right)$$

$\Rightarrow$

$$\frac{1}{\alpha_1(M_Z)} = \frac{1}{\alpha_1(M_{cut})} - \frac{Z_2}{2\pi} \ln Q \qquad \left(Q = \frac{M_{cut}}{M_Z}\right)$$

— A.1

and

$$\frac{1}{\alpha_2(M_Z)} = \frac{1}{\alpha_2(M_{cut})} - \frac{Z_1}{2\pi} \ln Q.$$

— A.2

Subtracting them gives:

$$\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)} = \frac{Z_1 - Z_2}{2\pi} \ln Q$$

— C

At M_{cut} $\alpha_1 = \alpha_2$ or $g_1 = g_2$ which we require

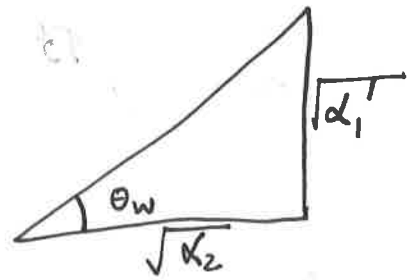
$* \quad g_2^2 = \dfrac{e^2}{\sin^2\theta_W}$ $\quad \boxed{g^2 = 4\pi\alpha}$

$$\Rightarrow \alpha_2 = \frac{\alpha_{EM}}{\sin^2\theta_W} \quad \left(\sin^2\theta_W = 0.231\right)$$

$$= 0.03357$$

then $\alpha_1' = (0.03357)\tan^2\theta$

$$= 0.0100$$



Using eq. ⓒ and setting the normalization $5/3$, we get,

$$\frac{1}{\alpha_1(M_Z)} - \frac{1}{\alpha_2(M_Z)} = \frac{Z_1 - Z_2}{2\pi}\ln(Q)$$

where $Q = \dfrac{M_{cutoff}}{M_Z}$.

$$\frac{1}{0.016} - \frac{1}{0.033} = \frac{19/6 + 81/20}{2\pi}\ln Q$$

$$e^{\left(\frac{(29.69)\,2\pi}{3.1666 + 4.05}\right)} = Q$$

$$\Rightarrow Q = e^{\frac{186.45}{7.2166}} \Rightarrow Q \simeq e^{25.8362}$$

But $$Q = \frac{M_{\text{scale required}}}{91 \text{ GeV}}$$

$$\Rightarrow \quad M_{\text{scale}}\Big|_{\substack{\text{at which} \\ g_1 = g_2}} \cong \frac{10^{13} \text{ GeV}}{(1.51 \times 10^{13} \text{ GeV})}.$$

So 10^{13} GeV is the scale at which $\underline{g_1 = g_2}$.

Let's use either of the equations to find (eq. A.1 or A.2)

$\alpha_1 (M_{\text{cut}})$

$$\frac{1}{0.016} = \frac{1}{\alpha_1 (M_{\text{cut}})} + \frac{(81/20)}{2\pi} \ln Q.$$

$$Q \cong e^{25.8362}$$

$$\ln Q = 25.8362$$

$$\Rightarrow \alpha_1 (M_{\text{cut}}) \approx 0.023$$

So,

$$\frac{1}{\alpha_s (M_Z)} = \frac{1}{0.023} - \frac{7}{2\pi} (25.8362)$$

$$\Rightarrow \quad \boxed{\alpha_s(M_Z) \cong 0.07}$$

Expected

$\cdot\ \alpha_s(M_Z) = 0.11$

The result will be independent of no. of families of fermions i.e N_g since all Z_i's have same contribution of $-\frac{4}{3}N_g$ and we encounter them as differences in our calculation.

2. $V(\Phi) = \frac{1}{2} m^2 \, Tr(\Phi^2) + \frac{1}{4} \lambda_1 \, Tr(\Phi^4) + \frac{1}{4} \lambda_2 \, (Tr\, \Phi^2)^2$ - (1)

$\Phi = v \, diag(\alpha_1 \ldots \alpha_5)$ ... (2)

$\sum_{i=1}^{5} \alpha_i = 0 \quad ; \quad \sum_{i=1}^{5} \alpha_i^2 = 1$

Rewriting eq. (1) above using (2) we get,

$$V(\Phi) = \frac{1}{2} m^2 v^2 \sum \alpha_i^2 + \frac{1}{4} \lambda_1 v^4 \sum \alpha_i^4 + \frac{1}{4} v^4 \lambda_2 \left(\sum \alpha_i^2\right)^2 \quad \ldots \text{(2A)}$$

$\frac{dV(\Phi)}{dv} = 0$ gives ;

$$\Rightarrow \quad m^2 v + [\lambda_1 X(\alpha) + \lambda_2 Y(\alpha)] v^3 = 0$$

where $X(\alpha) = \sum_i \alpha_i^4$

$Y(\alpha) = 1$

$$\Rightarrow v^2 = \frac{-m^2}{\lambda_1 X(\alpha) + \lambda_2 Y(\alpha)} \quad \ldots (3)$$

Using (3) in (2A) we get,

$$V(\Phi) = \frac{1}{2} m^2 v^2 \sum \alpha_i^2 + \frac{1}{4} \lambda_1 \frac{m^4 X(\alpha)}{[\lambda_1 X(\alpha) + \lambda_2 Y(\alpha)]^2} + \frac{1}{4} \frac{m^4 \lambda_2 Y(\alpha)}{[\lambda_1 X(\alpha) + \lambda_2 Y(\alpha)]^2}$$

$$= \frac{1}{2} m^2 \left(\frac{-m^2}{\lambda_1 X(\alpha) + \lambda_2 Y(\alpha)} \right) + \downarrow + \downarrow$$

$$= -\frac{1}{2} \frac{m^4}{\lambda_1 X(\alpha) + \lambda_2 Y(\alpha)} + \frac{1}{4} m^4 \frac{X(\alpha)\lambda_1 + \lambda_2 Y(\alpha)}{[X(\alpha)\lambda_1 + Y(\alpha)\lambda_2]^2}$$

$$= -\frac{1}{4} \frac{(m^2)^2}{\lambda_1 X(\alpha) + \lambda_2 Y(\alpha)} \quad \ldots (4)$$

b) To minimize $V(\Phi)$ means that we extremize $\lambda_1 X(\alpha) + \lambda_2 Y(\alpha)$, because then $V(\Phi)$ will get more negative. It means that we just ~~extremize~~ minimize $\boxed{\sum_i \alpha_i^4 = X(\alpha)}$ with constraints

Given also: $(\lambda_1, \lambda_2 > 0)$

$\sum_i \alpha_i^2 = 1$ and $\sum_i \alpha_i = 0$.

Since, we've two constraints, we use Lagrange's multipliers Λ_1 and Λ_2 and extremize

$$\sum_i \frac{1}{4} \alpha_i^4 + \frac{1}{2} \Lambda_1 \sum \alpha_i^2 + \Lambda_2 \alpha_i .$$

After differentiating w.r.t α_i, we get a cubic equation

$$\alpha_i^3 + \Lambda_1 \alpha_i + \Lambda_2 = 0 \quad \text{for each } \alpha_i$$

Also sum of roots of cubic equation $= \frac{-b}{a}$

And since here, we don't have any α_i^2, sum of roots is zero.

Let's base our argument now that only two values of α_i occur. Let's call them A_+ and A_-. Also, let's assume that $A_\pm$ occurs $N_\pm$ times where $N_+ + N_- = 5$.

Thus $\sum_i \alpha_i = N_+ A_+ + N_- A_- = 0$

$$\sum_i \alpha_i^2 = N_+ A_+^2 + N_- A_-^2 = 1$$

$$\Rightarrow \quad A_\pm^2 = \frac{N_\mp}{N_\pm 5} \quad \text{or} \quad A_\pm^4 = \frac{(N_\mp)^2}{25 (N_\pm)^2}$$

Also we can take $\delta > 0$ and $N_+ > N_-$ and write

$$\left.\begin{aligned} N_+ &= \frac{N}{2} + \delta \\ N_- &= \frac{N}{2} - \delta \end{aligned}\right\} N = 5$$

$$\begin{aligned}
\sum_i \alpha_i^4 &= N_+ A_+^4 + N_- A_-^4 \\
&= \frac{N_+ (N_-)^2}{25 (N_+)^2} + \frac{N_- (N_+)^2}{25 (N_-)^2} \\
&= \frac{(N_-)^2}{25 N_+} + \frac{(N_+)^2}{25 N_-} \\
&= \frac{(N_-)^3 + (N_+)^3}{25 N_+ N_-} = \frac{\left(\frac{N}{2} - \delta\right)^3 + \left(\frac{N}{2} + \delta\right)^3}{25 N_+ N_-}
\end{aligned}$$

$$= \frac{\frac{N^2}{4} + \frac{3}{2}\delta N^2}{25 N_+ N_-}$$

$$= \frac{\frac{25}{4} + \left(\frac{3}{2}\right) 25\delta}{25\left(\frac{N^2}{4} - \delta^2\right)} = \frac{\frac{25}{4} + \frac{75}{2}\delta}{25\left(\frac{25}{4} - \delta^2\right)}$$

$$= \frac{\frac{1}{4} + \frac{3}{2}\delta}{\frac{25}{4} - \delta^2} = \frac{1 + 6\delta}{25 - 4\delta^2}$$

$\Sigma \alpha_i^4$ increases as <u>δ increases</u>. We need to pick smallest possible positive δ.

$$N_+ = \frac{N}{2} + \delta$$

$$= 2.5 + \delta$$

and $N_- = 2.5 - \delta$

Clearly $\boxed{\delta_{min} = 0.5}$ here and that gives

$$\boxed{N_+ = 3} \; ; \; \boxed{N_- = 2}$$

We can now follow the same calculation of $\Sigma \alpha_i^4$ for case where we have three possible values of α_i i.e A_+, A_- and A_0.

But the constraint that $\sum_i \alpha_i^4$ should be as ~~less~~ small as possible (minima) will rule out this case of A_+, A_- & A_0. Hence, we conclude that given $SU(5)$ breaks as:

$$SU(5) \xrightarrow[\text{minimum}]{\text{at}} SU(3) \otimes SU(2) \otimes U(1)$$

and $\langle \Phi \rangle$ is given by

$$\langle \Phi \rangle = N v \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \quad \ldots \quad (5)$$

where N is some extra normalization, we will fix.

Clearly $\sum_i \alpha_i = 0$ for (5)

$$\sum_i \alpha_i^2 = 12 + 18$$

$$= 30 \ \checkmark.$$

Hence $N = \sqrt{\frac{1}{30}}$

This gives, $\langle \Phi \rangle = \frac{v}{\sqrt{30}} \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$

c) $SU(5)$ has $5^2-1 = 24$ generators and $SU(3)\otimes SU(2)\otimes U(1)$ has $8+3+1 = 12$ unbroken generators.

Thus, 12 of the generators of $SU(5)$ become massive.

The gauge boson mass is calculated through the kinetic term

$$\mathcal{L}_{kin} = Tr\left((\mathcal{D}_\mu \Phi)(\mathcal{D}_\mu \Phi^\dagger)\right)$$

$$= Tr\left((\partial_\mu \Phi + g[A_\mu, \Phi])^\dagger(\partial_\mu \Phi + g[A_\mu, \Phi])\right)$$

$$\simeq g^2 A^a_\mu A^{\mu b}\, Tr\left[[T^a,\langle\Phi\rangle][T^b,\langle\Phi\rangle]\right] \quad \left(A^\mu = A^\mu_a T^a\right)$$

where T^a and T^b are generators (broken ones).

The reason other 12 generators are not important for spectrum of gauge bosons calculation is because, $\langle\phi\rangle$ commutes with those generators and trace is 0 $\Rightarrow \boxed{m=0}$ massless.

Commuting generators correspond to massless gauge field.

In this breaking, 12 massless fields are there.

Let's consider a traceless $SU(5)$ generator that does not commute with $\langle\phi\rangle$ and give a mass for the bosons.

$$t = \frac{1}{2}\begin{pmatrix} & & & & -i \\ & & & & \\ & & & & \\ & & & & \\ i & & & & \end{pmatrix}_{5\times5} \in SU(5)$$

and $$\langle\phi\rangle = \frac{v}{\sqrt{30}}\begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

$$[t, \langle\phi\rangle] = \frac{v}{2\sqrt{30}}\begin{pmatrix} & & & & -5i \\ & & & & \\ & & & & \\ & & & & \\ 5i & & & & \end{pmatrix}$$

$$\mathrm{Tr}\,\left|[t, \langle\phi\rangle]\right|^2 = \frac{v^2}{4(30)}\,50$$

$$= \frac{5v^2}{12}$$

But

$$\mathcal{L}_{\text{mass term}} = \frac{1}{2} m_{\text{boson}}^2 A^2$$

$$\mathcal{L}_{\text{kin}} = g^2 \left(\frac{5}{12} v^2 \right) A^2$$

Comparing we get

$$m^2_{\text{Gauge boson}} = \frac{5 v^2 g^2}{6} \checkmark .$$

Similarly, we can do for other broken generators of $SU(5)$.

3. Four degenerate Dirac fermions in four Euclidean dimension.

Global symmetry : $\underline{SO(4)_{\text{Euclidean}} \otimes SU(4)_{\text{flavor}}}$.

Also note that $SO(4)_f$ is a sub-group of $SU(4)_{\text{flavor}}$.**

We can concentrate on $\underline{SO(4)_E \otimes SO(4)_{\text{flavor}}}$ symmetry to make the selection of <u>diagonal subgroup</u> later more obvious.

Under the aforementioned flavor & Euclidean symmetries, the fermions transform as

$$\Psi_{\alpha i} \longmapsto \xi_{\alpha\beta}\, \Psi^{\beta j}\, \chi^T_{ji}$$

where α, β ~~is~~ are the euclidean index, and i, j are flavor indices.

If we do an analogous procedure to twisting and identify $\begin{matrix}\alpha \to i \\ \beta \to j\end{matrix}$, then

$$\boxed{\Psi \rightarrow \xi \Psi \xi^T} \quad \ldots \quad (1)$$

Ψ behaves like a $\underline{4 \times 4}$ matrix and transforms according to (1) above.

** Subgroup of flavor symmetry should coincide with $SO(4)_{\text{Euclidean}}$ **

The subgroup under which fermions can be regarded as 4×4 matrix is clearly:

$$\rightarrow \text{Diagonal}\left(SO(4)_E \otimes SO(4)_{flavor}\right)$$

<u>To construct the action</u> out of these $\underline{4\times 4}$ matrices Ψ, γ etc., we'll surely have to trace over them.

$$S = \int d^4x \left\{ \mathrm{Tr}\left(\bar{\Psi}\gamma^\mu \partial_\mu \Psi\right) - m\mathrm{Tr}\left(\bar{\Psi}\Psi\right)\right\} \quad \cdots (2)$$

where Ψ and γ^μ are $\underline{4\times 4}$ matrices !!

In the massless limit, (2) becomes

$$S = \int d^4x \ \mathrm{Tr}\left(\bar{\Psi}\gamma^\mu \partial_\mu \Psi\right) \quad \cdots (3)$$

We show that in $\underline{m=0}$ case, the action in (3) breaks into two independent pieces. We denote,

$$\Psi_+ = P_+\Psi = \frac{1}{2}\left(\Psi + \gamma_5 \Psi \gamma_5\right) \quad \cdots (4)$$

$$\Psi_- = P_-\Psi = \frac{1}{2}\left(\Psi - \gamma_5 \Psi \gamma_5\right) \quad \cdots (5)$$

By abuse of notation, let's denote Ψ_- as Ψ_L and Ψ_+ as ** Ψ_R. Then,

Dirac adjoint $\left(\bar{\Psi} = \psi^\dagger \gamma^0\right)$

[Note that $(\gamma^5)^\dagger = \gamma^5$]

$\{\gamma^5, \gamma^0\} = 0$

$$
\begin{aligned}
\bar{\Psi}_L &= \Psi_L^\dagger \gamma^0 \\
&= \frac{1}{2}\left(\psi^\dagger - \gamma_5 \psi^\dagger \gamma_5\right)\gamma^0 \\
&= \frac{1}{2}\left(\psi^\dagger \gamma^0 + \gamma_5 \bar{\Psi} \gamma_5\right) \\
&= \frac{1}{2}\left(\bar{\Psi} + \gamma_5 \bar{\Psi} \gamma_5\right) \quad \cdots \quad (6)
\end{aligned}
$$

and similarly,

$$
\bar{\Psi}_R = \frac{1}{2}\left(\Psi - \gamma_5 \Psi \gamma_5\right) \quad \cdots \quad (7)
$$

If the action in (3) has to break into two independent pieces, we can have following combinations.

$$
\underbrace{Tr\left(\bar{\Psi}_+ \gamma_\mu \partial_\mu \Psi_-\right)}, \quad \underbrace{Tr\left(\Psi_- \gamma_\mu \partial_\mu \Psi_+\right)}
$$

or

$$
Tr\left(\bar{\Psi}_\pm \gamma_\mu \partial_\mu \Psi_\pm\right)
$$

We go back to old Ψ_+ and Ψ_-

$\Psi_+ \longmapsto \Psi_R$

$\Psi_- \longmapsto \Psi_L$

We'll show now that underlined terms only become part of Eq. (3)

** To not confuse "$\dagger$" (dagger) with "+" etc. **

A better argument can be also given on the basis of fact that $P_-^2 = P_-$; but $P_+ P_- = P_- P_+ = 0$.

We'll try to write out $Tr(\bar{\Psi}_+ \gamma_\mu \partial_\mu \Psi_-)$ explicitly.

$$Tr(\bar{\Psi}_+ \gamma_\mu \partial_\mu \Psi_-) = \frac{1}{2}(\bar{\Psi} - \gamma_5 \Psi \gamma_5)\gamma_\mu \partial_\mu \frac{1}{2}(\Psi + \gamma_5 \Psi \gamma_5)$$

$$= \frac{1}{2} Tr(\bar{\Psi} \gamma_\mu \partial_\mu \bar{\Psi})$$

$$= \frac{1}{2} Tr(\bar{\Psi} \gamma^\mu \partial_\mu \bar{\Psi}) \quad \ldots (8)$$

$$\left\{ \begin{array}{l} Tr(\gamma^5 (\text{odd no. of } \gamma^\mu)) = 0 \\ (\gamma^5)^2 = \mathbb{1} \end{array} \right\}$$

Similarly,

$$Tr(\bar{\Psi}_- \gamma^\mu \partial_\mu \Psi_+) = \frac{1}{2} Tr(\Psi \gamma^\mu \partial_\mu \Psi) \quad \ldots (9)$$

Using (8) & (9) in (3), we get

$$S = \int d^4x \left\{ Tr(\bar{\Psi}_+ \gamma^\mu \partial_\mu \Psi_-) + Tr(\bar{\Psi}_- \gamma^\mu \partial_\mu \Psi_+) \right\}$$