Some notes about ABJM

Abstract: These are some notes in progress on the ABJM model, its integrability, and how it is related to the low-energy limit of three-dimensional maximally supersymmetric YM. I hope to type my 20-30 pages of notes about these in the near future.

Contents

1 Introduction

ABJM model after the work by Aharony, Bergman, Jafferis and Maldacena [1] is a duality between AdS_4/CFT_3 which was inspired by earlier works by Bagger and Lambert and others. According to this duality, the large N limit of a 3d superconformal SU(N) $\times$ SU(N) Chern-Simons theory with level k [1] is dual to M-theory on $AdS_4 \times \mathbb{S}_7/\mathbb{Z}_k$. We have to take the limit k, N $\to \infty$, with the 't Hooft coupling $\lambda = N/k$ being fixed. The global symmetry group in this case is Osp(6|4) which is orthosymplectic group of rank 5. The field content is given by two gauge fields, four complex scalars (Y^A) and four Weyl spinors (ψ^A) Magnon dispersion relation for integrable spin chain of $\mathcal{N} = 4$ SYM which is due to the underlying SU(2|2) symmetry in both cases is given by,

$$\epsilon(p) = \sqrt{Q^2 + 4h^2(\lambda)\sin^2(p/2)} \tag{1.1}$$

For AdS_5/CFT_4, $h(\lambda)$ is simply $\sqrt{\lambda}/4\pi$ which is related to S-duality in some sense. Since, for AdS_4/CFT_3 we have no notion of this duality, a uniform function is not expected. Indeed, it was found that the weak and strong coupling limits are given by $h(\lambda) = \lambda(1+A\lambda^2)$ and $h(\lambda) = \sqrt{\lambda}/2 + B + \cdots$. It is expected that one can replace all occurences of λ by $h(\lambda)$ to go from AdS_5/CFT_4 to AdS_4/CFT_3. For the cases where $k = 1, 2$, we have an enhancement of the supersymmetry from $\mathcal{N} = 6$ to $\mathcal{N} = 8$ (and a corresponding SO(8) R-symmetry). ABJM in case of $k = 1$ reduces to the theory of N M2-branes in flat space, and for $k = 2$ reduce to the theory of N M2-branes on $R_8/\mathbb{Z}_{k=2}$. Also, in these cases there is an enhancement of the global symmetry group from the usual Osp(6|4) to Osp(8|4). However, we will mostly never discuss these since the model is expected to be integrable only when k is large ('t Hooft limit). There is no evidence of relation between $\mathcal{N} = 6$ ABJM and $\mathcal{N} = 4$, SYM_4, the AdS_4/CFT_3 correspondence seems to be another instance where integrability plays an important role in the gauge/string duality

We can define, $\lambda = g_{CS}^2 N = \frac{N}{k}$. Instead of working with ABJM model, one can consider ABJ model which is generalization to $U(M)_k \times U(N)_{-k}$. In this case, one often defines, $\hat{\lambda} = N/k, \lambda = M/k$. Unlike $\mathcal{N} = 4$ spin-chain, ABJM spin chain is alternating because the matter fields are in the bi-fundamental representation of the product gauge group. It might look something like, $\mathrm{Tr}(Y^1 Y_4^\dagger Y^1 Y_4^\dagger \cdots$. A choice for the vacuum with BPS operators can be one mentioned above. The choice of the vacuum breaks the Osp(6|4) down to SU(2|2) $\times$ U(1) which then becomes the symmetry group of the spin chain.

[1] Actually, the first gauge group has a Chern-Simons action at level k and the second has level -k

C_{123} for the 3-point function of length-two chiral primary operators (CPO) in planar ABJ(M) theory at weak 't Hooft coupling was calculated in [2].

The ground states of the spin chains in both theories correspond to CPOs. The dimensions of these operators are protected by supersymmetry and thus have vanishing anomalous dimension. A reasonable choice for a spin chain vacuum in $\mathcal{N}=4$ is $\mathrm{Tr}(\mathrm{Z^L})$, while in the ABJM model a convenient choice as mentioned above is $\mathrm{Tr}(Y^1 Y_4^\dagger Y^1 Y_4^\dagger \cdots) = \mathrm{Tr}(Y^1 Y_4^\dagger)^L$.

AdS_5/CFT_4	AdS_4/CFT_3
$\mathcal{N}=4$ SYM $\Longleftrightarrow$ Type IIB on $AdS_5 \times S_5$	$\mathcal{N}=6$ Chern-Simons matter $\Longleftrightarrow$ Type IIA on $AdS_4 \times \mathbb{CP}^3 (= \mathbb{S}^7/\mathbb{Z}_k)$
$T_{\text{string}} = \frac{L^2}{\alpha'} = \sqrt{\lambda}$	$T_{\text{string}} = \frac{R^2_{AdS}}{\alpha'} = 2^{7/2}\pi^2\sqrt{\lambda}$
$\lambda = g^2 N$, $g_s = g^2/4\pi = \frac{\lambda}{4\pi N}$	$\lambda = N/k$, $g_s = \frac{2(2\pi^2)^{1/4}\lambda^{1/4}}{k}$
SU(N) gauge group	$U(N)_k \times U(N)_{-k}$
$\mathfrak{psu}(2,2\|4)$	$\mathfrak{Osp}(6\|4)$

Table 1. The comparison between two most studied dualities featuring SCFTs.

For ABJM – $N, k \to \infty$ keeping λ fixed. Planar, or 't Hooft, limit which is given by — $\mathcal{N}=4$ SYM – $N \to \infty$ keeping $\lambda = g^2 N$ fixed.

For $k \to \infty$ with N/k fixed the theory is compactified to ten dimensions and reduces to type IIA string theory in $AdS_4 \times \mathbb{CP}^3$. This limit has been extensively studied and corresponds to the ?t Hooft limit in the CFT, where N is large with $\lambda = N/k$ fixed. If k is $\mathcal{O}(1)$ and N is large, then the dual is eleven-dimensional M=-theory.

BMN like operators in ABJM is different since they correspond to excited states of membranes rather than strings in the case of $\mathcal{N}=4$ SYM.

$\mathbb{CP}^3$ has an isometry group SU(4) which is the same as that of $\mathcal{N}=4$ SYM.

1.1 Sectors of $\mathcal{N}=4$ SYM

- SU(2) sector – This contains two complex scalar fields (Z, X) and is often dubbed the simplest one.

- SL(2) sector – This contains one complex scalar fields (Z) with one covariant derivative and is the simplest non-compact sector. The states can generally be written as, $|\mathcal{D}^I ZZZZZ\cdots\rangle$.

- SU(1|1) sector – This contains one complex scalar fields (Z) with one fermionic field (Ψ). This is the simplest which contains both bosons and fermions.

- SU(3|2) sector – This contains three complex scalar fields (Z, Y, X) with one fermionic field (Ψ). This is the simplest which contains both bosons and fermions.

- PSU(1,1|2) – This contains two complex scalar fields (Z,X) with a fermionic field (Ψ) and its conjugate ($\overline{\Psi}$) This is the simplest which contains both bosons and fermions.

References

[1] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, "N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals," *JHEP* **10** (2008) 091, arXiv:0806.1218 [hep-th].

[2] D. Young, "ABJ(M) Chiral Primary Three-Point Function at Two-loops," *JHEP* **07** (2014) 120, arXiv:1404.1117 [hep-th].