$\mathcal{L}$, S, and $\mathcal{H}$'s IN PHYSICS

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★ $\mathcal{N}=4$ SYM - (3+1)-dimensions

$$\mathcal{L} = \frac{N}{\lambda}\mathrm{Tr}\left[F^{\mu\nu}F_{\mu\nu} + (D_\mu X_i)^2 - \frac{1}{2}[X_i, X_j]^2 + \Psi^T \not{D}\Psi + \Psi^T\gamma_i[X,\Psi]\right] \tag{1}$$

BFSS (0+1)-dimensions

$$\mathcal{L} = \mathrm{Tr}\Bigg((D_t X_i)^2 - [X_i, X_j]^2\Bigg) + \Psi^T \not{D}\Psi + \Psi^T\gamma_i[X,\Psi] \tag{2}$$

BMN/PWMM - (0+1)-dimensions with i,j,k $= 1\cdots 3$ and $M = 4\cdots 9$

$$\mathcal{L} = \mathcal{L}_{\mathrm{BFSS}} + \mathrm{Tr}\left[\left(\frac{\mu}{3}X_I\right)^2 + \left(\frac{\mu}{6}X_M\right)^2 + \frac{\mu}{4}\Psi^T_\alpha\gamma^{123}_{\alpha\beta}\Psi_\beta + \frac{\sqrt{2}\mu}{3}\epsilon_{IJK}X_I X_J X_K\right]. \tag{3}$$

SYK in (0+1)

$$\mathcal{H} = \frac{1}{4!}\sum_{i,j,k,l=1}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l \tag{4}$$

IKKT (0+0)-dimensions, with i,j $= 1\cdots 10$

$$\mathcal{L} = \mathrm{Tr}\Big([X_I, X_J]^2\Big) + \Psi^T \not{D}\Psi + \Psi^T\gamma_I[X,\Psi] \tag{5}$$

Einstein-Hilbert action

$$S = \frac{c^4}{16\pi G}\int d^4x\,(R - 2\Lambda) \tag{6}$$

Nambu-Goto (NG) action

$$S = -T \int d^2\sigma \sqrt{-(\dot{X}^2)(X')^2 + (\dot{X} \cdot X')^2} \tag{7}$$

$\dot{X}^\mu = \partial X^\mu / \partial \tau$, $(X')^\mu = \partial X^\mu / \partial \sigma$

Polyakov action

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \tag{8}$$

Chern-Simons action

$$S = \frac{k}{4\pi} \int d^3x \; \epsilon^{\mu\nu\rho} \; \mathrm{Tr} \left(A_\mu \partial_\nu A_\rho - \frac{2i}{3} A_\mu A_\nu A_\rho \right) \tag{9}$$

Principal Chiral Field

$$\mathcal{L} = \frac{\beta}{2} \mathrm{Tr} \left(\partial_\mu g^{-1} \partial_\mu g \right) \tag{10}$$

Massless Schwinger (1+1)

$$\mathcal{L} = \frac{1}{2} (\epsilon^{\mu\nu} \partial_\nu A_\mu)^2 - e j^\mu A_\mu + \overline{\Psi} \partial\!\!\!/ \Psi \tag{11}$$

▲ Massive Thirring

$$\mathcal{L} = \overline{\Psi} \partial\!\!\!/ \Psi - m_F \overline{\Psi} \Psi - \frac{g}{2} (\overline{\Psi} \gamma^\mu \Psi)^2 \tag{12}$$

O(N) non-linear σ in 1+1

$$\mathcal{L} = \frac{1}{2g} \sum_{i=1}^{N} (\partial^\mu \hat{n}_i)^2 \tag{13}$$

Ising

$$\mathcal{H} = -\sum_{ij} J_{ij} \sigma_i \sigma_j - \sum_j h_j \sigma_j \tag{14}$$

▲ <u>SINE-GORDON</u>

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^4}{\lambda}\left(1 - \cos\left(\frac{\sqrt{\lambda}\phi}{m}\right)\right) \tag{15}$$

★ <u>HEISENBERG MODEL [1928, SOLVED BY BETHE (1931)]</u>

$$\mathcal{H}_{XXX} = \frac{J}{2}\sum_L \left(\sigma_L^x\sigma_{L+1}^x + \sigma_L^y\sigma_{L+1}^y + \sigma_L^z\sigma_{L+1}^z\right) \tag{16}$$