The simplified version of (9) is :

$$S = \int d\tau \left[i \left(\frac{d\phi}{d\tau} + s \frac{\partial V}{\partial \phi} \right) B + \frac{1}{2} B^2 - i \bar{\psi} \left(\frac{d}{d\tau} + s \frac{\partial^2 V}{\partial \varphi \partial \varphi} \right) \psi \right] \quad \text{— (A1)}$$

We could eliminate the B from action but an advantage of retaining it is that the supersymmetry is nilpotent and so reminiscent of BRST symmetry. We have the following transformation rules:

$$\{Q, \phi\} = \psi \ ; \ \{Q, \psi\} = 0 \ ; \ \{Q, \bar{\psi}\} = B \ , \ \{Q, B\} = 0$$
$$\text{and } \{Q, Q\} = 0$$

The bosonic part of the action is clearly minimized by the first order equation

$$\frac{d\phi}{d\tau} + s \frac{\partial V}{\partial \phi} = 0 \quad \cdots\cdots \quad (A)$$

which means that in the "steepest descent" (one-loop) approximation only such paths will contribute. These classical path are called "instantons" & the action vanishes for these configurations.

For a general theory, the sparing argument (realize that the classical instanton path is point ..) shows that the absolute minima of the action are:

$$\frac{d\phi^i}{d\tau} = 0 \quad , \quad s \frac{\partial V^i}{\partial \varphi^i} = 0 .$$

If $s \neq 0$, the relevant points are the critical point of V, or on other hand, when $s=0$, all the points of the target manifold enter.

When R_{ijkl} of eq. (9) can be ignored, (i.e when the tangent (target) manifold is $\mathbb{R}^n$), the Nicolai map

$$\xi^i = \frac{d\varphi^i}{d\tau} + s\, g^{ij}(\varphi) \frac{\partial V}{\partial \varphi^j}$$

is as before, such that the Jacobian of the map cancels the absolute value of the fermionic Pfaffian.

* <u>Langevin Approach</u>

We can trivialize the theory with the use of Nicolai Map. We can now discuss a method for creating the theory from same map. This relies on the notion of a Langevin equation. An equation of the form:

$$\xi = \frac{d\phi}{d\tau} + s\frac{\partial V}{\partial \phi} \quad \ldots \quad (B)$$

is known as Langevin equation and the method developed here is called Langevin approach. The time that enters in (B) is a stochastic time variable, but here is just taken as real time. Let's start with a trivial Gaussian action:

$$S_0 = \frac{1}{2}\oint d\tau \left(G - \xi(\phi)\right)^2$$

where $\xi(\phi)$ is Nicolai Map. It is clear that we could easily shift G and eliminate any dependence on φ

Then we would be left with Gaussian integration over the G' field, but also unweighted sum over φ. This is similar to situation in gauge theories. We use gauge invariance to fix the gauge and then maintaining BRST symmetry after the process.

The problem then reduces to identification of gauge invariance of the action, obtaining BRST symmetry & then choosing an appropriate gauge condition. Action is invariant under:

$$\delta\phi = \lambda \quad ; \quad \delta G = \frac{\partial \xi}{\partial \phi} \lambda \quad \ldots \quad (C)$$

Carrying out BRST Quantization of above, we get the partition f^n as

$$Z = \int_\phi e^{-S(\varphi)} \Delta_{FP} \quad \checkmark.$$

Let's now see how eq. (C) can be turned to BRST symmetry. (Let $\lambda \to \psi$)

$$\{Q, \phi\} = \psi$$

$$\{Q, G\} = \left(\frac{\partial \xi}{\partial \phi}\right) \psi$$

$$\{Q, \psi\} = 0$$

$$\{Q, Q\} = 0$$

To this we must add anti-ghost $\bar{\Psi}$ and a Lagrange multiplier (auxillary field ; B)

$$\{Q, \bar{\Psi}\} = B$$

$$\{Q, B\} = 0 \qquad \text{still} \quad \underline{Q^2 = 0}.$$

Gauge fixed action is then :

$$S = \oint d\tau \left[\frac{1}{2}(G - \xi)^2 + i\{Q, \bar{\Psi} G\} \right]$$

$$= \oint d\tau \left[\frac{1}{2}(G - \xi)^2 + iBG - i\bar{\Psi}\left(\frac{d}{d\tau} + s\frac{\partial^2 W}{\partial\phi\partial\phi}\right)\Psi \right]$$

$$= \oint d\tau \left[\frac{1}{2}G'^2 + iG'B + i\left(\frac{d\phi}{d\tau} + s\frac{\partial W}{\partial\phi}\right)B \right] \left(\begin{array}{l}\text{integrate over } B, \\ \text{shift } G\end{array}\right)$$

This when integrated over G' yields the action of form A1. Thus starting with a gaussian action over random field, we have produced a SUSY QM model !!

Now, if instead of A1, our aim was to get to Eq. 9, we had to use a generalized gaussian action like :

$$S_0 = \frac{1}{2}\oint g_{ij}(\phi)\, K^i K^j$$

where

$$K^i = G^i - \frac{d\phi^i}{d\tau} - s\, g^{ij}(\phi)\frac{\partial V(\phi)}{\partial\phi^j}$$

$$= G^i - \xi^i$$

This is invariant under following transformations:

$$\delta\phi^i = \lambda^i \quad ; \quad \delta G^i = \frac{\partial z^i}{\partial \phi^j}\lambda^j - \Gamma^i_{jk} K^j \lambda^k$$

Now turning the equation above ~~to~~ BRST symmetry as before is not trivial as target space $\neq \mathbb{R}^n$ and commutator of infinitesimal transformations do not close when acting on G^i

$$\left[\delta(\lambda_2), \delta(\lambda_1)\right] G^i = \lambda_1^j \lambda_2^k R^i_{\ell jk} K^\ell$$

$R_{ijk\ell}$ was zero and we could do gauge fixing & retain BRST symmetry but here it is not closed and we need Batalin-Vilkovisky procedure to obtain a gauge fixed action with transformation rules such that Eq. 9 form is achieved.