Aspects of Classical and Quantum Computing of Quantum Many-Body Systems

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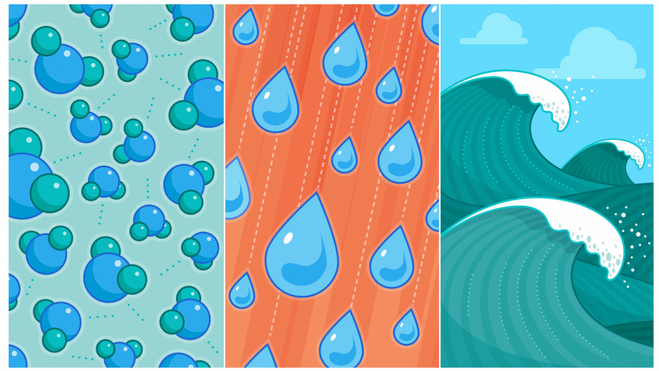
rgjha.github.io

Outline

- Introduction to tensor RG (renormalization group)
- Applications to (classical) Ising model with magnetic field, $2d$ $O(2)$ model. Then move to $3d$!
- Another facet of tensors: Real-time evolution in Ising Field Theory (IFT) using Matrix Product States (MPS) of (quantum) Ising model.
- Moving to quantum computing: Qubits ($d = 2$ Hilbert space), Qumodes (continuous variables [CV], infinite dimensional HS)
- Application of qubit method to understand $O(3)$ model recently by other groups and our ongoing work on CV formulation.

Different RG methods



Various renormalization group (RG) schemes (list not exhaustive) have been introduced over the past 5-6 decades:

- Kadanoff's spin blocking RG [1966] & Wilson's RG [1975]
- Density Matrix Renormalization Group (DMRG) [White, 1992]
  (DMRG is an extension to Wilson RG and is well-suited to all 1d systems not only restricted to impurity problems)
- Tensor Renormalization Group [Levin and Nave, 2007] + HOTRG [Xie et al., 2012]
- Tensor Network Renormalization (TNR) [Vidal et al., 2015]

Figure: Quanta Magazine

But why tensors?

- Provides an arena to study lower-dimensional (critical and gapped) systems faster than any other known method available today! [2d classical Ising model in 15 seconds on a laptop]
- Formulation in terms of tensors can help us study models where the usual Monte Carlo (MC) methods fail (such as finite-density, θ-term). In addition, the thermodynamic limit can be approached faster and partition function can be computed unlike MC.
- Description of a quantum state in terms of tensors (MPS) can be useful to study real-time dynamics
- Known to play a key role in emergence of space-time [understanding holography]

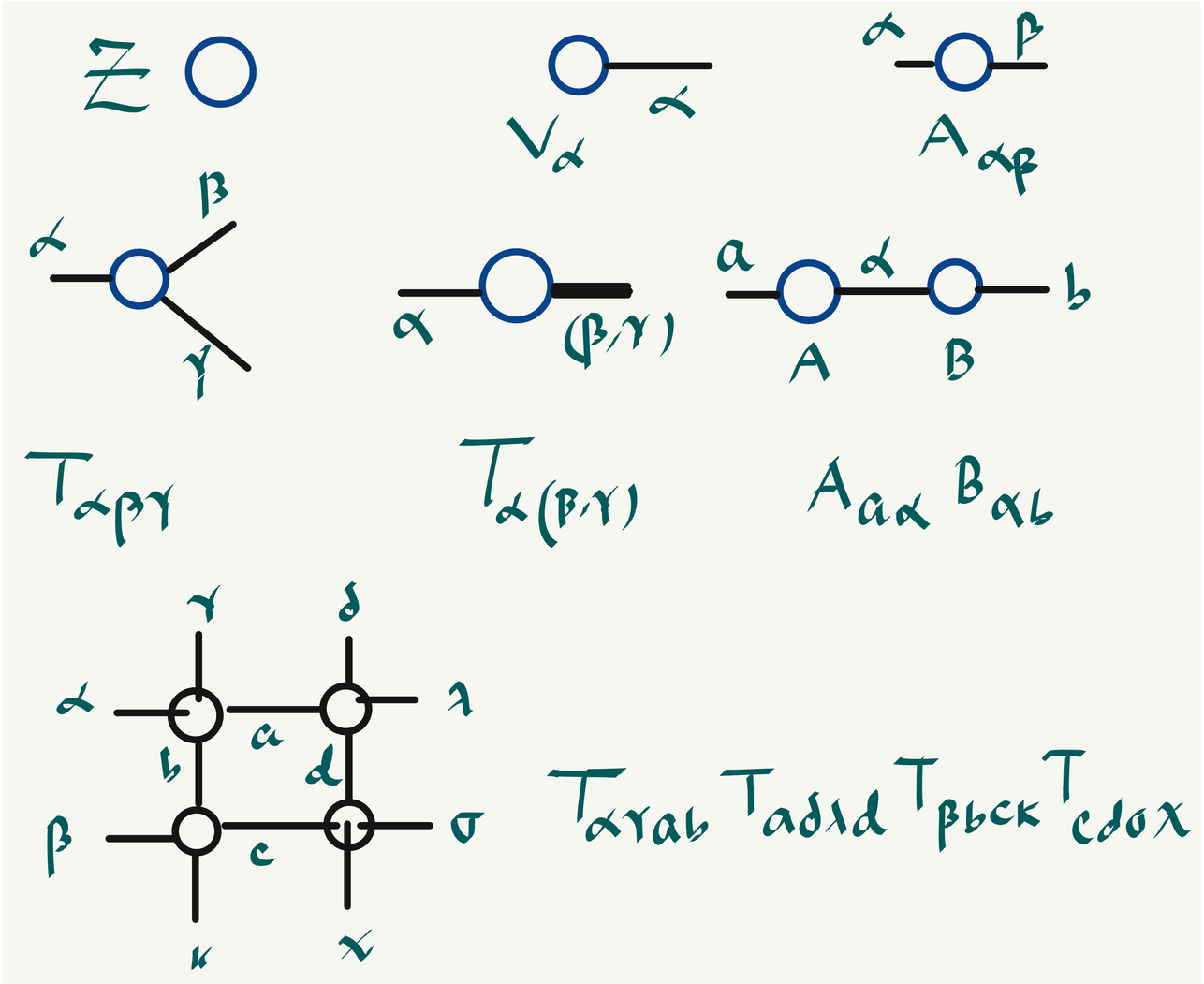
Formulation of tensors

Tensor methods have both Lagrangian and Hamiltonian applications.

- State approach: We can approximate the ground state i.e., $|\psi\rangle = \sum_{i_1,\cdots,i_N} C_{i_1,\cdots,i_N} |i_1,\cdots,i_N\rangle$ of a model with local Hamiltonian of N spins in fewer coefficients than 2^N, O(N).

- Action approach: We approximate the partition function using tensor networks considering decomposition of Boltzmann weight (truncate if necessary) and then coarse-graining by performing successive iterations.

Notation


Z
V α
α
A αβ
α
β
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α
(β,γ)
a
α
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T αβγ
T α(β,γ)
A aα B αb
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λ
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c
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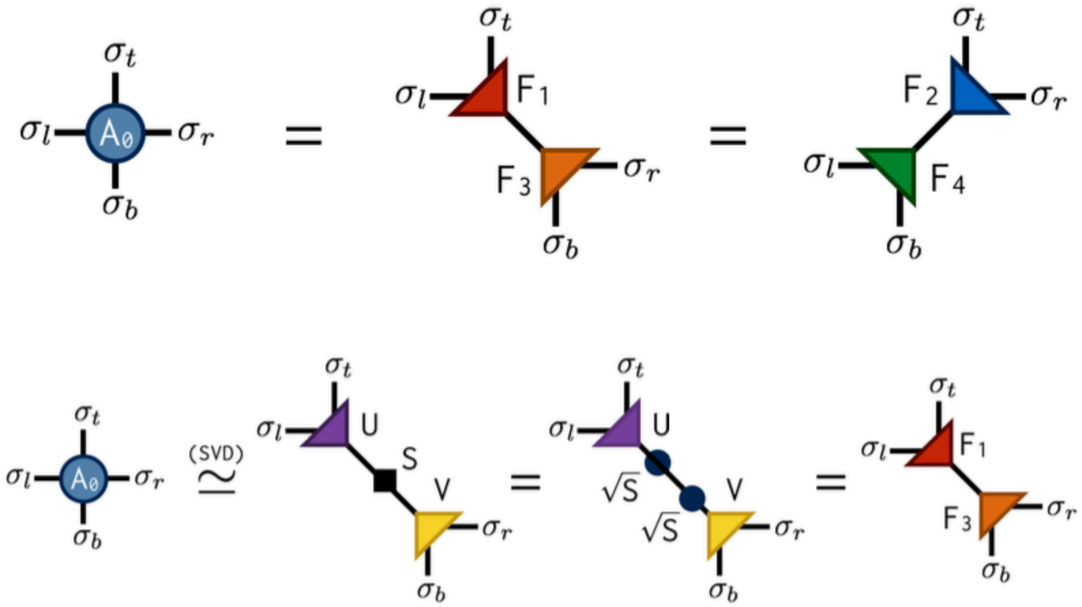
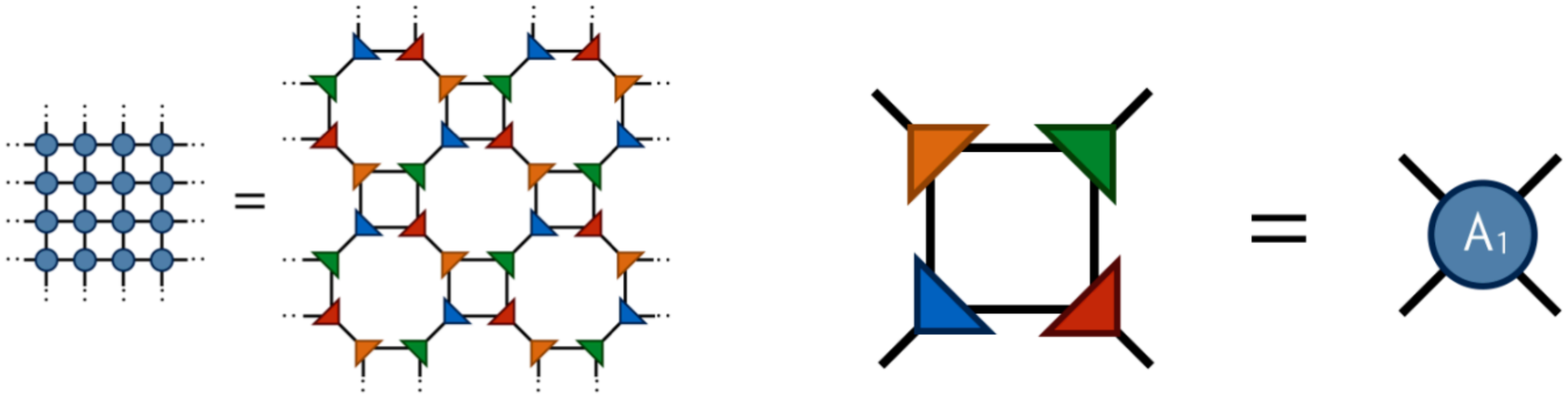
χ
T αγab T adλd T βbcκ T cδσχ

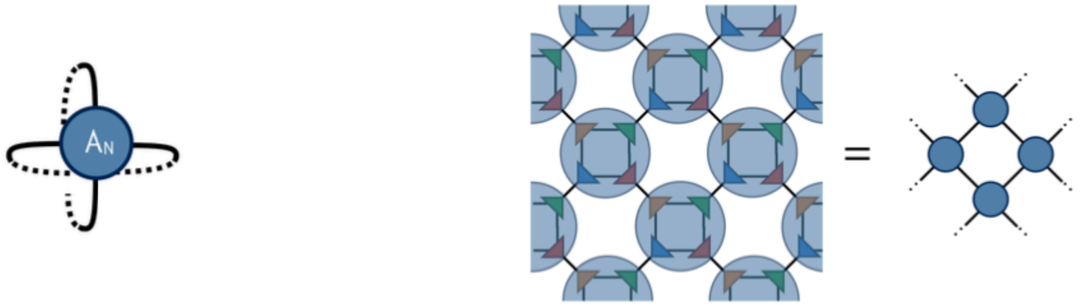
This talk [TRG]!

For this talk we will restrict to the application of tensor networks when dealing with statistical systems in Euclidean dimensions. This amounts to evaluating Z to best possible accuracy. This problem usually belongs to NP (non-polynomial) complexity class! We will start with an initial network and then perform coarse-graining to approach the correct target theory with best approximation. For example, the schematic representation of TRG can be shown as:



TRG continued!





Improved TRG

In its crude form as developed by Levin and Nave in 2007, this method cannot deal with higher dimensional systems. For that, after about five years, HOTRG [higher-order] TRG was developed based on higher-order SVD (HOSVD) to reduce the errors due to truncation. First introduced in arXiv: 1201.1144 and has been successfully applied to statistical systems in $d = 3,4$.

Coarse-graining renormalization by higher-order singular value decomposition

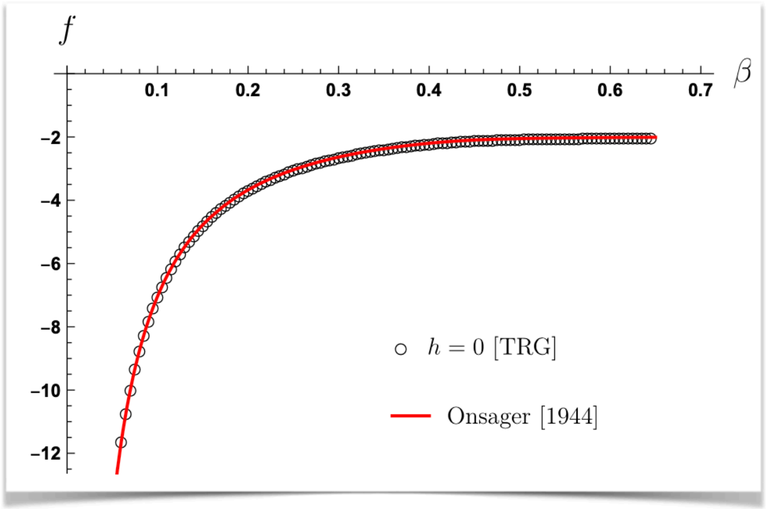
Z. Y. Xie, J. Chen, M. P. Qin, J. W. Zhu, L. P. Yang, T. Xiang

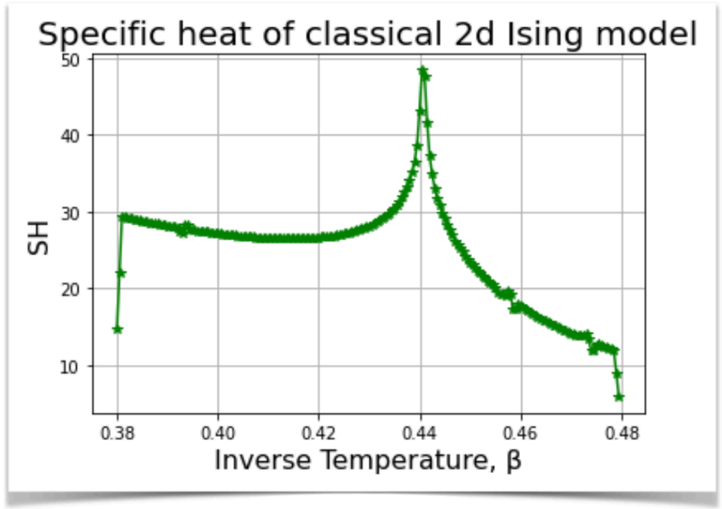
We propose a novel coarse graining tensor renormalization group method based on the higher-order singular value decomposition. This method provides an accurate but low computational cost technique for studying both classical and quantum lattice models in two- or three-dimensions. We have demonstrated this method using the Ising model on the square and cubic lattices. By keeping up to 16 bond basis states, we obtain by far the most accurate numerical renormalization group results for the 3D Ising model. We have also applied the method to study the ground state as well as finite temperature properties for the two-dimensional quantum transverse Ising model and obtain the results which are consistent with published data.

Simple demonstration

We have motivated this idea of TRG but it is best if we apply it to some simple system with known solution. Ising model is the perfect playground for this! The exact solution is given by [Onsager, 1944] with critical inverse temperature $\beta \approx 0.440687$

$$f(\beta) = -\frac{1}{\beta}\left(\ln(2) + \frac{1}{8\pi^2}\int_0^{2\pi}\int_0^{2\pi} \ln\left[2\cosh^2(2\beta) - \sinh(2\beta)\cos(\phi_1) - \sinh(2\beta)\cos(\phi_2)\right] d\phi_1 d\phi_2 \right)$$





15 seconds on modern laptop!

Ising with magnetic field

But, if we introduce magnetic field, the model becomes unsolvable. It is an outstanding open problem for more close to 80 years! Some cases for imaginary magnetic field values are solvable due to Yang-Lee [1952] and Merlini [1974] but for general real h, not much is known on a regular lattice. For random graph, it was solved by Kazakov and Boulatov in 1986 by a map to Hermitian two-matrix model. If we define $z = e^{-2\beta h}$, then Onsager case is $z = 1$ while Merlini solution is for $z = -1$

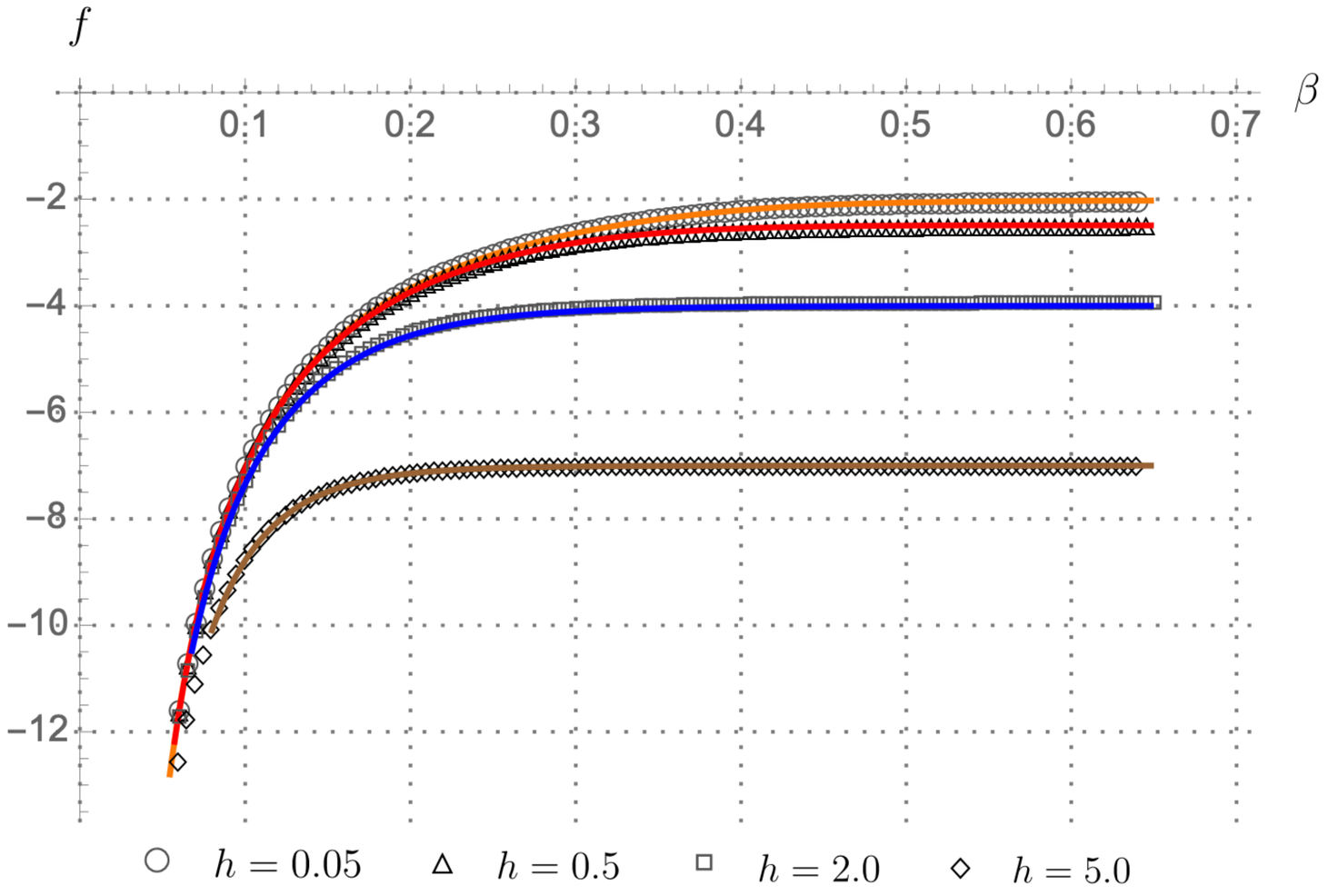
$$f\left(\beta, \frac{i\pi}{2\beta}\right) = -i\frac{\pi}{2} - \frac{1}{\beta}\left(\ln 2 + \frac{1}{16\pi^2}\int_0^{2\pi}\int_0^{2\pi} \ln\left[\sinh^2(2\beta)\left(1 + \sinh^2(2\beta) + \frac{\cos(\phi_1+\phi_2) - \cos(\phi_1-\phi_2)}{2}\right)\right] d\phi_1 d\phi_2\right).$$

On the solution of the two-dimensional ising model with an imaginary magnetic field $\beta H = h = i\pi/2$

D. Merlini

Lettere al Nuovo Cimento (1971-1985) **9**, 100–104 (1974) | Cite this article

Classical Ising with magnetic field - Numerics



O(2) model

RGJ, arXiv: 2004.06314

We can study the simplest spin model with continuous O(2) global symmetry using these methods. It was studied first in 2013 by Yu et al. [1309.4963] and by Vanderstraeten et al. [1907.04576]. We revisit this work and improved the results by few digits of precision for determination of the BKT phase transition. The Hamiltonian is given by:

$$\mathscr{H} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j) - h \sum_i \cos\theta_i$$

In order to construct the tensor representation, we decompose the Boltzmann weight using Jacobi-Anger expansion and integrate over θ-variables:

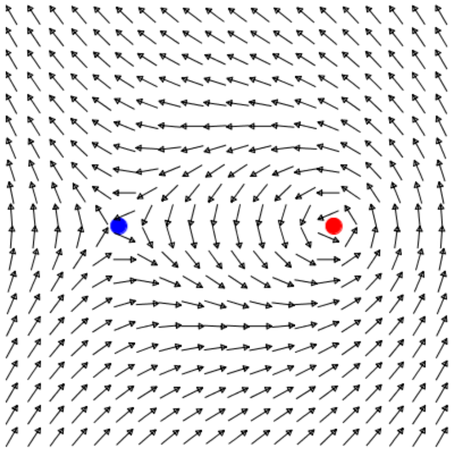
$$\exp\Big(\beta \cos(\theta_i - \theta_j)\Big) = I_0(\beta) + \sum_{\nu=-\infty,\neq 0}^{\infty} I_\nu(\beta) e^{i\nu(\theta_i - \theta_j)}$$

$$T_{ijkl} = \sqrt{I_i(\beta) I_j(\beta) I_k(\beta) I_l(\beta)}\, I_{i+k-j-l}(\beta h)$$

O(2) model

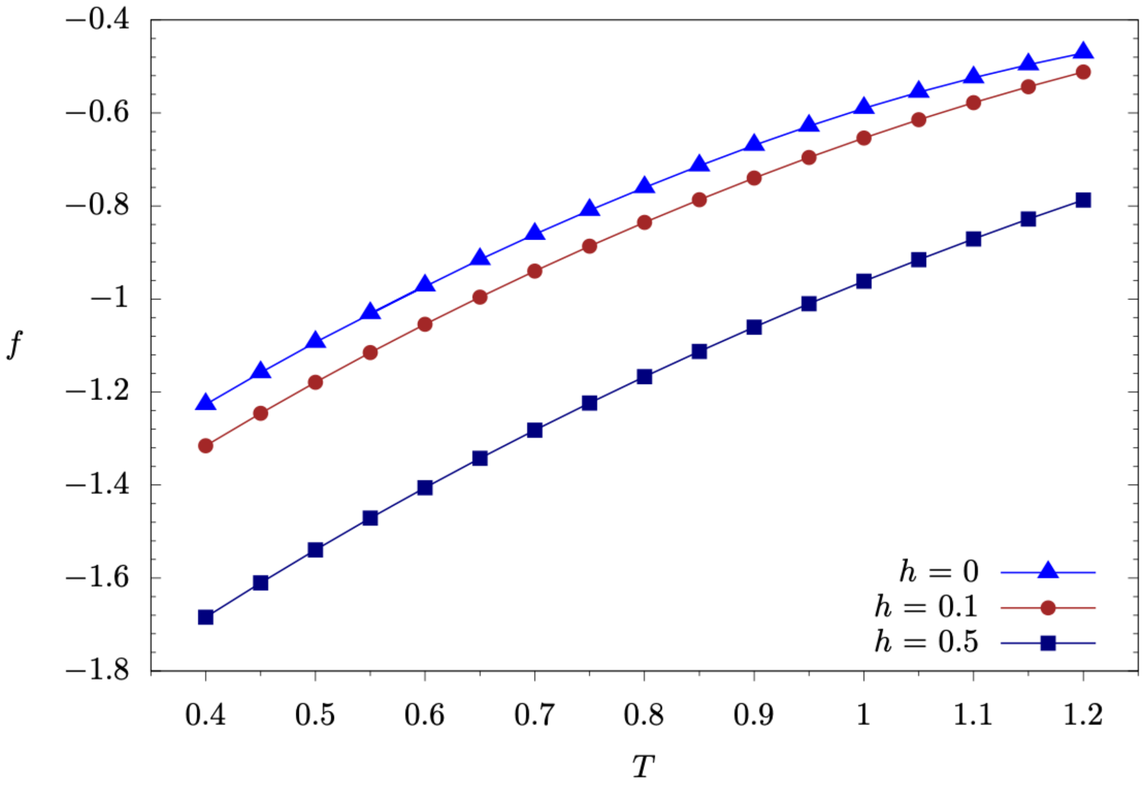
RGJ, arXiv: 2004.06314

The δ-function for $h = 0$ ensures the conservation of U(1) charges. This model has a famous BKT transition corresponding to unbinding of vortex pairs. Note that in two dimensions, continuous symmetry cannot break due to the famous Mermin-Wagner -Hohenberg-Coleman theorem and hence one might expect no phase transition but the BKT transition is special case. The transition is from a quasi-long range ordered (QLRO) to a disordered phase. At some temperature, all the vortices and anti-vortices are free to move, which destroys the correlations between distant spins and breaks QLRO. It was the first example of a topological phase transition. It is of infinite order in Ehrenfest classification sense - “none of the derivatives of free energy is discontinuous”.



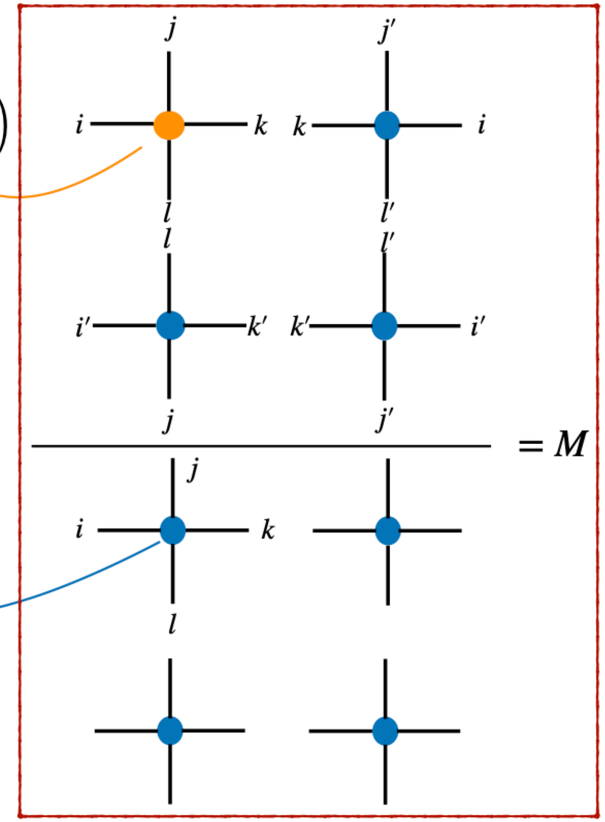
Results - O(2) model

RGJ, arXiv: 2004.06314



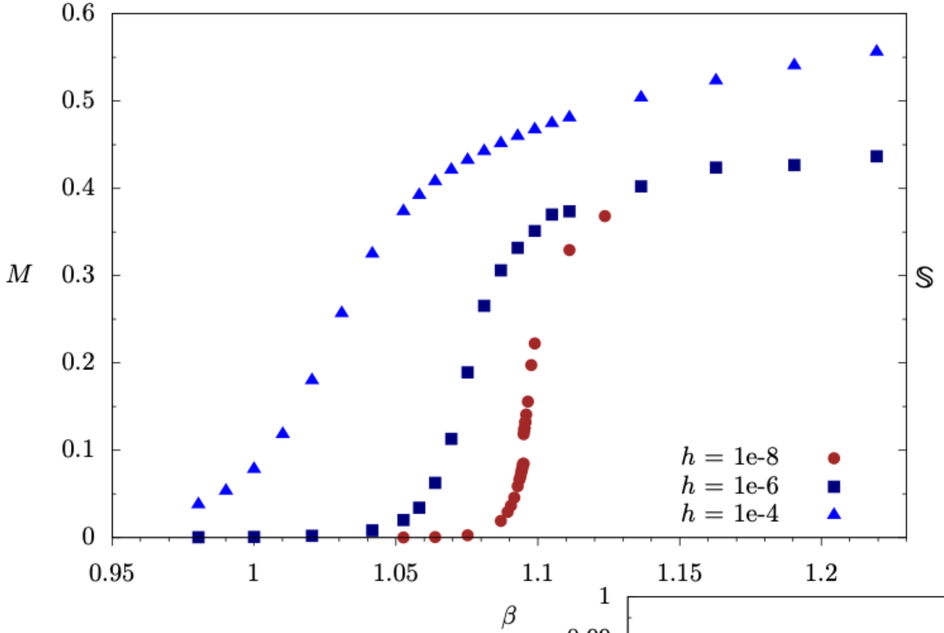
$$\tilde{\mathrm{T}}_{\mathrm{ikjl}} = \sqrt{I_i(\beta)I_k(\beta)I_j(\beta)I_l(\beta)}\left(\frac{I_{i+j-k-l-1}(\beta h) + I_{i+j-k-l+1}(\beta h)}{2}\right)$$

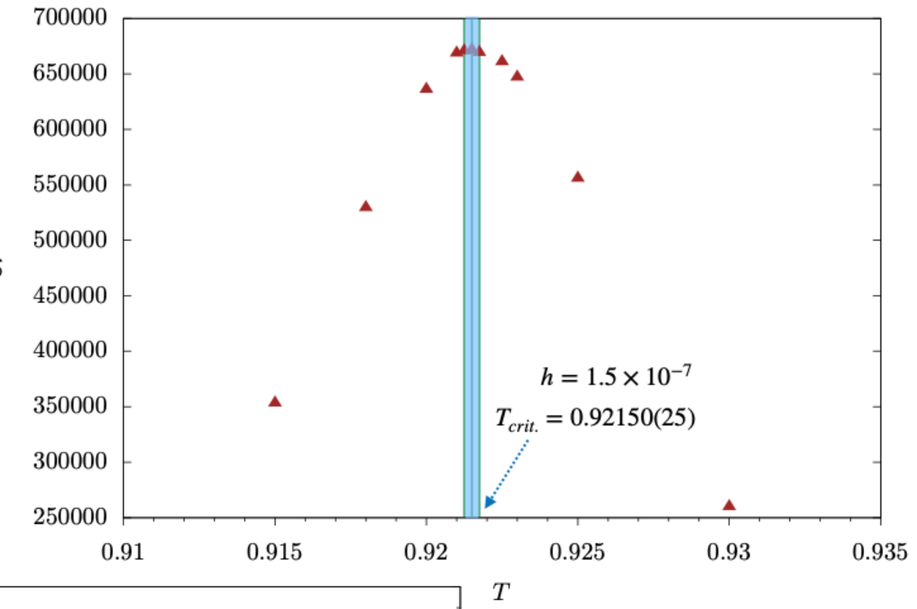
$$\mathrm{T}_{\mathrm{ikjl}} = \sqrt{I_i(\beta)I_k(\beta)I_j(\beta)I_l(\beta)}\, I_{i+j-k-l}(\beta h)$$

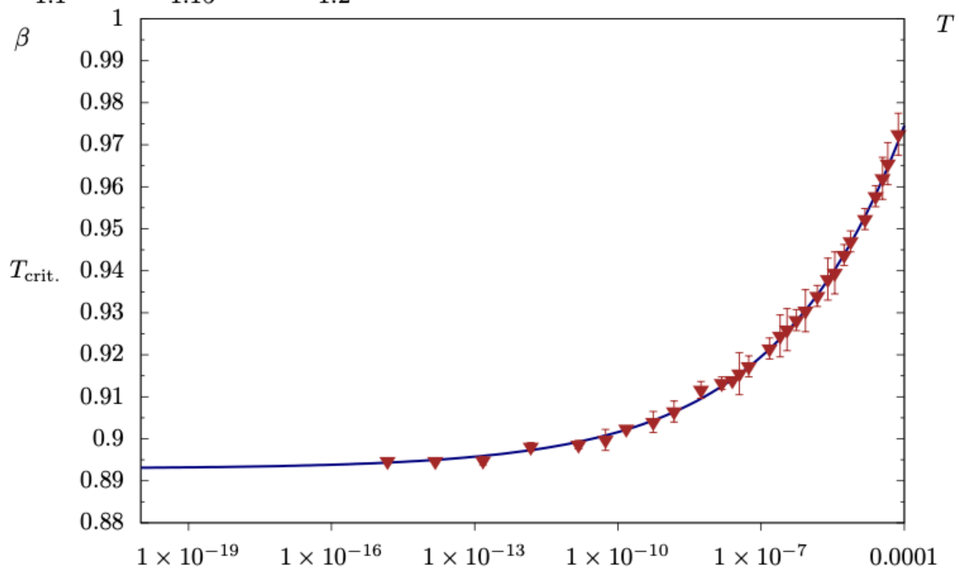


Results - O(2) model

RGJ, arXiv: 2004.06314

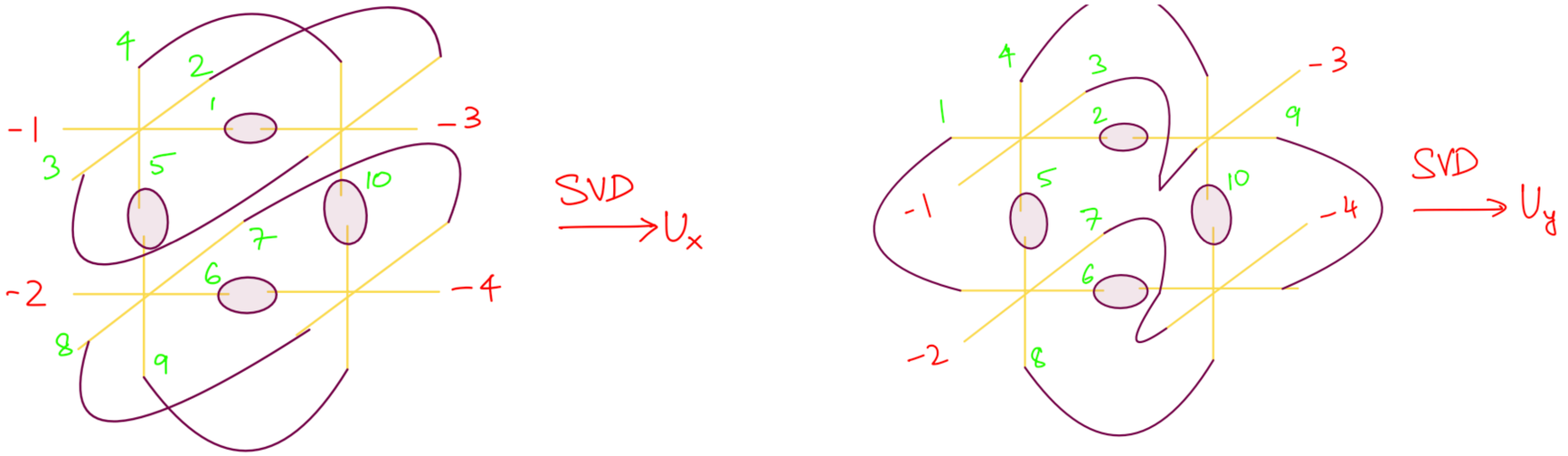






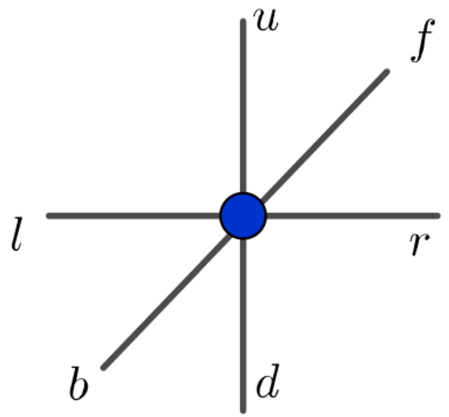
Moving to higher dimensions

Though tensor methods works very well for lower-dimensional systems, it was not explored much for $d \geq 3$ because of several problems involved (computer time which scales like $O(D^{4d-1})$, memory requirements, effects of truncation etc.).

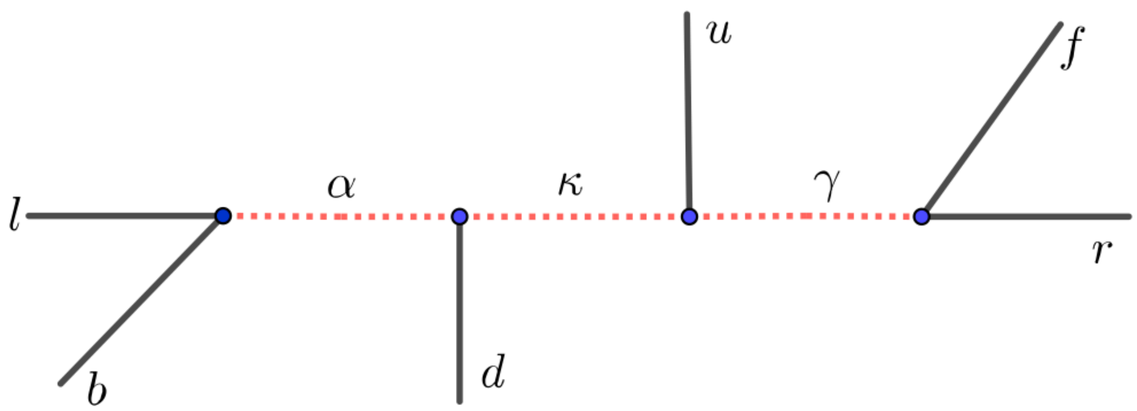


Triad method

In 2019, it was found that it is often faster to deal with not the rank-six tensor in 3d directly, but decompose it in terms of several rank-three tensor known as triads. This reduces the cost drastically and we can study some statistical systems which were more difficult before.

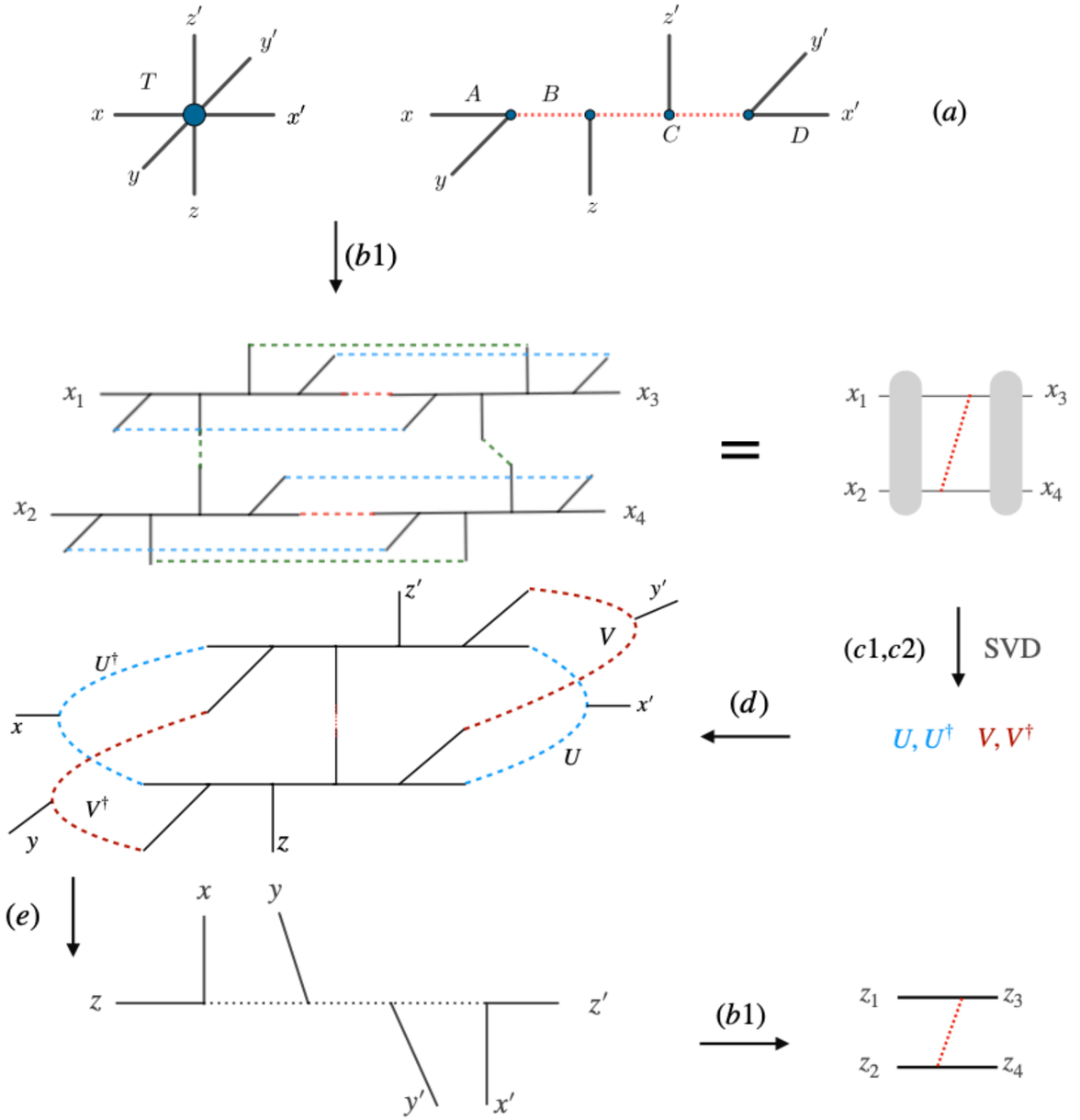


T_{lrudfb}



$A_{lb\alpha}B_{\alpha d\kappa}C_{\kappa u\gamma}D_{\gamma fr}$

Basic step in Triad TRG



Status of 3d spin systems [w tensors]

Apart from Ising model on cubic lattice, not much had been done for Potts model or even the O(2) model.

- Ising model studied but critical exponents not yet computed!
- q-state Potts model in the large q limit [RGJ, arXiv: 2201.01789]
- $O(2)$ model at finite number density [First study: RGJ, Bloch, Lohmayer, Meister, arXiv: 2105.08066]

Real-time scattering in Ising field theory (IFT)

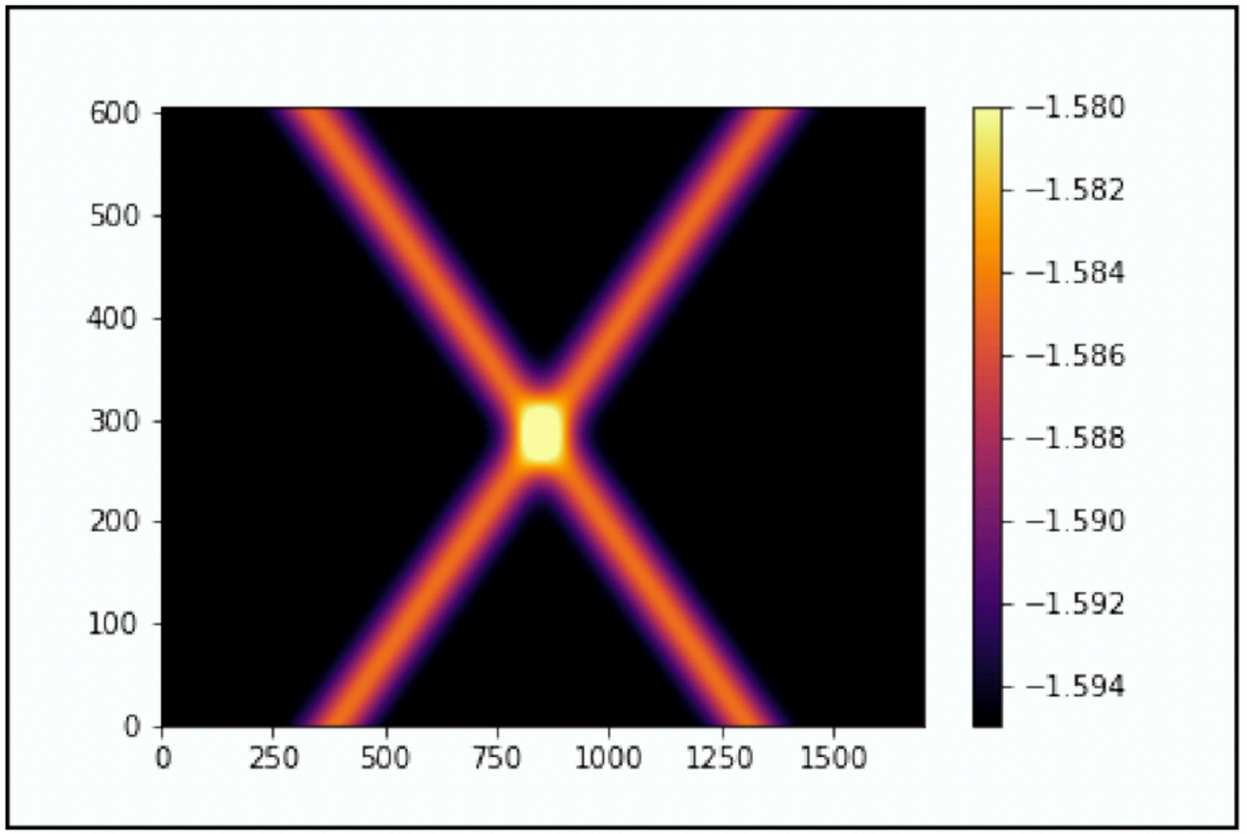


Figure Credits: Ashley Milsted, Dominik Neuenfeld

MPS approach to scattering in IFT

With tensor network methods, we can approximate the ground state of quantum Ising chain with local Hamiltonian $H = -\sum_{i=1}^{N} Z_i Z_{i+1} - hZ_i + gX_i$ where we take the double scaling limit $h \to 0,\ g \to 1$ (corresponding to critical temp in 2d classical case). If we define $\tau = T/T_C - 1$, then the RG parameter $\eta = \tau/|h|^{8/15}$ determines the behaviour of the model. Zamolodchikov found that the model is integrable for $\eta = 0$ and he computed the mass spectrum which consists of eight particles (three below the threshold of $2m$) and five owing their stability to integrability. This model also has another integrable limit of $\eta \to \infty$ for all τ. We first start with a random MPS and through imaginary-time evolution, find the ground state of the H given above.

Stable particles in $h - \tau$ plane

in progress, Milsted et al.

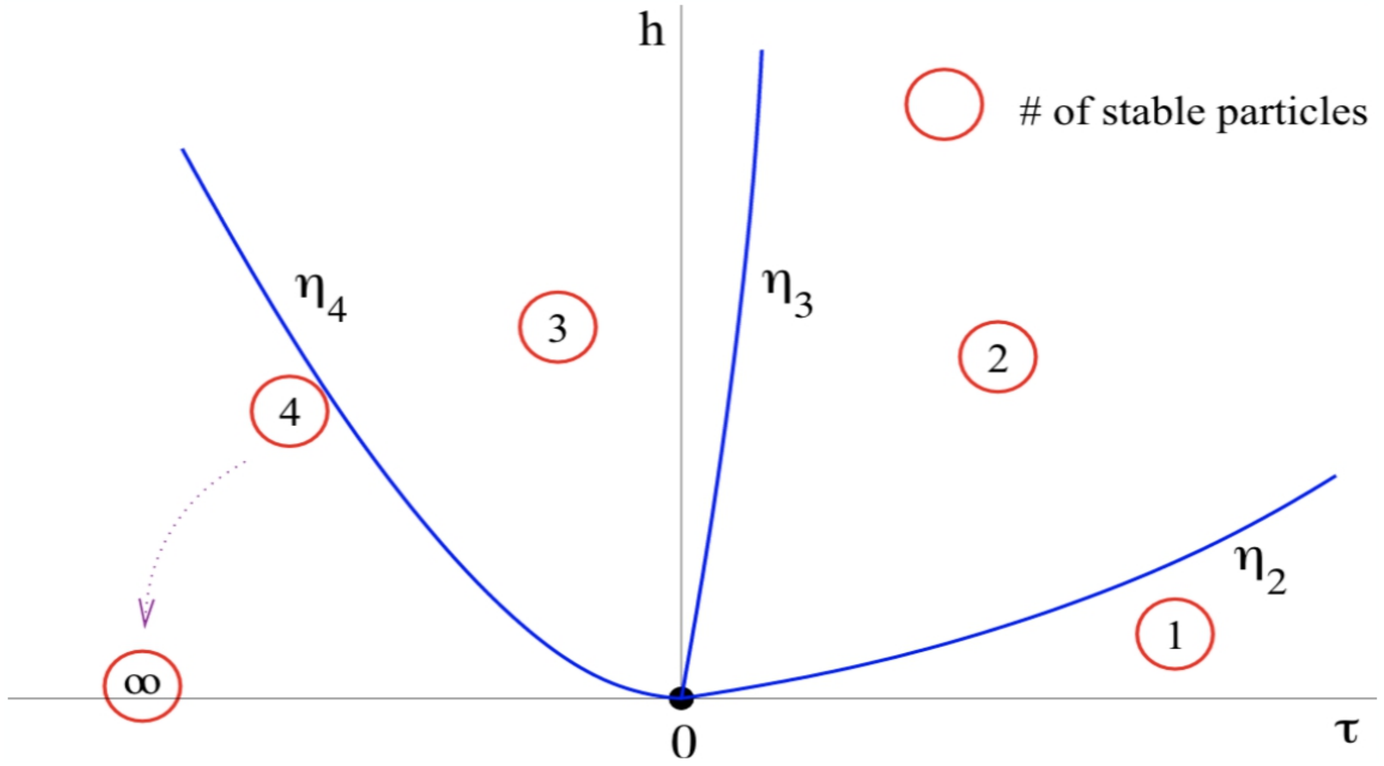


Figure Credits: G. Delfino
Delfino, Mussardo et al, arXiv: hep-th/0507133

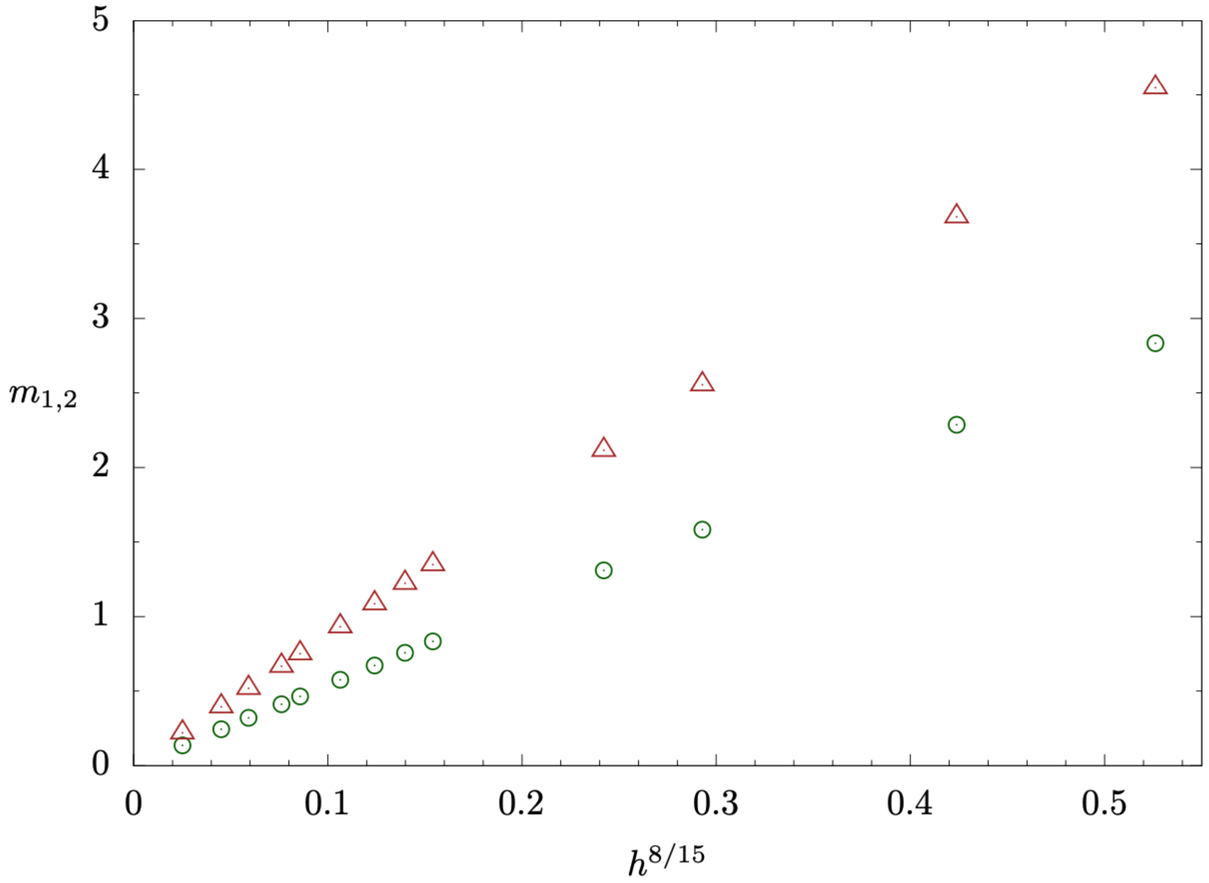
Spectrum at E_8 point

$$
\begin{aligned}
m_2 &= 2\cos\tfrac{\pi}{5} m_1 && \approx 1.618 m_1 \\
m_3 &= 2\cos\tfrac{\pi}{30} m_1 && \approx 1.989 m_1 \\
m_4 &= 2\cos\tfrac{\pi}{5}\cos\tfrac{7\pi}{30} m_1 && \approx 2.405 m_1 \\
m_5 &= 4\cos\tfrac{\pi}{5}\cos\tfrac{2\pi}{15} m_1 && \approx 2.956 m_1 \\
m_6 &= 4\cos\tfrac{\pi}{5}\cos\tfrac{\pi}{30} m_1 && \approx 3.218 m_1 \\
m_7 &= 8(\cos\tfrac{\pi}{5})^2\cos\tfrac{7\pi}{30} m_1 && \approx 3.891 m_1 \\
m_8 &= 8(\cos\tfrac{\pi}{5})^2\cos\tfrac{2\pi}{15} m_1 && \approx 4.783 m_1
\end{aligned}
$$

Zamolodchikov's solution is the most complicated integrable model known in Physics’’ — Subir Sachdev

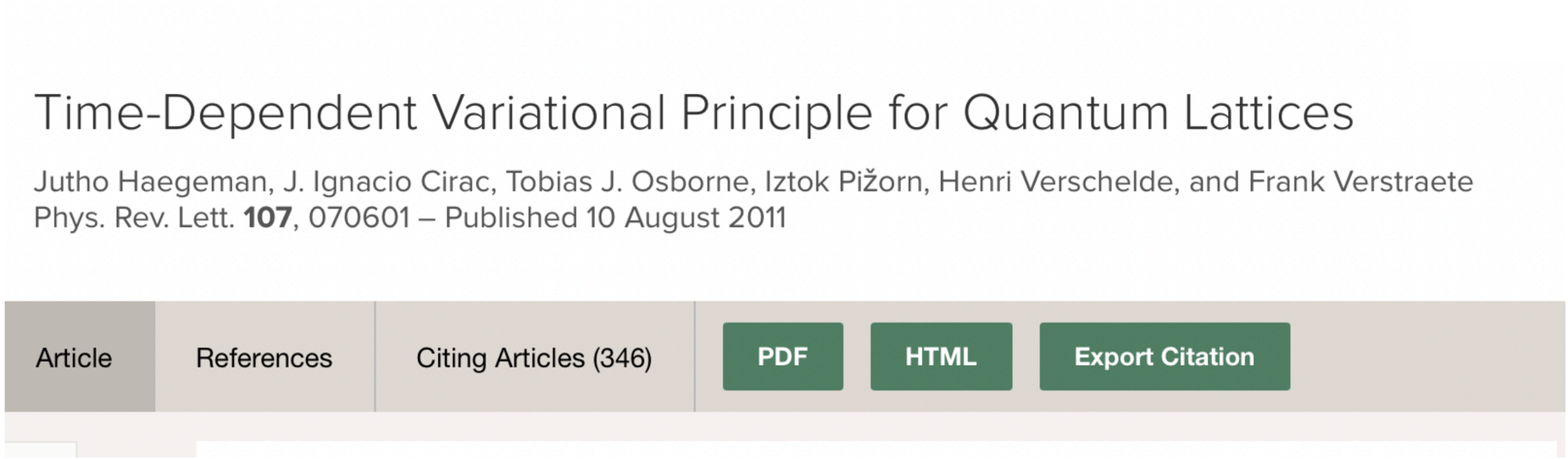
Spectrum close to E_8 point

All stable particles have very specific dependence on h and we checked this using MPS calculations. One can also compute η_2 and η_3 i.e., where the particle 2 and particle 3 becomes unstable. The data is at T = Tc.



Real-time evolution and scattering

Once we have created a MPS [does not always work!] which is faithful representation of the ground state of quantum spin chain, we construct excitations on top [quasi particles] and then evolve them in real-time using TDVP methods [time-dependent variational principle]. TDVP is a very popular alternative to Trotterization with several advantages and was introduced in the seminal paper:



History: Dirac-Frenkel-McLachlan in 1930s

Open questions!

- We know that $|S| = 1$ at two integrable points. However, we do not know how it behaves (interpolates) between these two limits. We can compute these close to the integrable points by doing form-factor perturbation theory (known as FFPT) but general regime needs support from numerical results. We know that close to FF, $P_{11} = 1 - P_{\text{prod.}} = 0$ till $E = 3m$, and $P_{\text{prod.}} > 0$ for $E > 3m$. We see this in our results and also the agreement to FFPT.
- What is the high-energy behaviour between this integrable points. There is a conjecture that $P_{11} = 1 - P_{\text{prod.}} \to 0$ as $E \to \infty$ close to FF and $P_{11} = 1 - P_{\text{prod.}} \to 1$ as $E \to \infty$ close to E8 with a transition between. We see some signs for this behaviour. There are other major 'complex' issues close to E8 (resonance etc.).

Note that in $d = 2$, we can have scattering without particle production (elastic) but it is prohibited in $d > 2$ by Aks theorem [S. Aks, "Proof that scattering implies production in quantum field theory," Journal of Mathematical Physics (1965), 516-532.]

Quantum computers

1981: Feynman gives a seminar at MIT in the summer and observes that it is impossible to simulate an evolution of a quantum system on a classical computer in an efficient way. He proposes a basic model for a quantum computer that would be capable of such simulations. Around the same time Toffoli introduces the reversible doubly-controlled quantum NOT gate.

1980: Manin publishes his paper (in Russian) and proposes the idea of quantum computer for the first time. See text and references for detail.

1976: Polish physicist Ingarden shows that Shannon information theory cannot be directly generalized to the quantum case because in the usual quantum mechanics of closed systems there is no general concept of joint and conditional probability. He showed, however, that it is possible to construct a quantum information theory which is a generalization of Shannon's theory.

1975: Poplavskii publishes 'Thermodynamical models of information processing' (in Russian) and shows the impractical computational nature of simulating quantum systems on classical computers.

1973: Holevo publishes paper showing that n qubits cannot carry more than n classical bits of information. This was a surprising result. Around same time, Bennett showed that computation can be done reversibly.

1969/1970: Wiesner discusses with Bennett his idea on conjugate coding which he argued can be used for printing bank notes (quantum money) that would be impossible to counterfeit. The exact date of the idea is not known since the published version is much later in 1983. Wiesner tried to publish it earlier by submitting to the IEEE Transactions on Information Theory, but, it was rejected.

1986

Quantum Mechanical Computers

By Richard P. Feynman

Introduction

This work is a part of an effort to analyze the physical limitations of computers due to the laws of physics. For example, Bennett[1] has made a careful study of the free energy dissipation that must accompany computation. He found it to be virtually zero. He suggested to me the question of the limitations due to quantum mechanics and the uncertainty principle. I have found that, aside from the obvious limitation to size if the working parts are to be made of atoms, there is no fundamental limit from these sources either.

We are here considering ideal machines; the effects of small imperfections will be considered later. This study is one of principle; our aim is to exhibit some Hamiltonian for a system which could serve as a computer. We are not concerned with whether we have the most efficient system, nor how we could best implement it.

Since the laws of quantum physics are reversible in time, we shall have to consider computing engines which obey such reversible laws. This problem already occurred to Bennett[1], and to Fredkin and Toffoli[2], and a great deal of thought has been given to it. Since it may not be familiar to you here, I shall review this, and in doing so, take the opportunity to review, very briefly, the conclusions of Bennett[2], for we shall confirm them all when we analyze our quantum system.

It is a result of computer science that a universal computer can be made by a suitably complex network of interconnected primitive elements. Following the usual classical analysis we can imagine the interconnections to be ideal wires carrying one of two standard voltages representing the local 1 and 0. We can take the primitive elements to be just two, NOT and AND (actually just the one element NAND = NOT AND suffices, for if one input is set at 1 the output is the NOT of the other input). They are symbolized in Fig. 1, with the logical values resulting on the outgoing wires, resulting from different combinations of input wires.

From a logical point of view, we must consider the wires in detail, for in other systems, and our quantum system in particular, we may not have wires as such. We see we really have two more logical primitives, FAN OUT when two wires are connected to one, and EXCHANGE, when wires are crossed. In the usual computer the NOT and NAND primitives are implemented by transistors, possibly as in Fig. 2.

What is the minimum free energy that must be expended to operate an ideal computer made of such primitives? Since, for example, when the AND operates the output line, c' is being determined to be one of two values no matter what it was before the entropy change is $ln(2)$ units. This represents a heat generation of $kT\,ln(2)$ at temperature T. For many years it was thought that this represented an absolute minimum to the quantity of heat per primitive step that had to be dissipated in making a calculation.

The question is academic at this time. In actual machines we are quite concerned with the heat dissipation question, but the transistor system used actually dissipates about $10^{10}kT$! As Bennett[3] has pointed out, this arises because to change a wire's voltage we dump it to ground through a resistance; and to build it up again we feed charge, again through a resistance, to the wire. It could be greatly reduced if energy could be stored in an inductance, or other reactive element.

Richard P. Feynman is a professor of theoretical physics at California Institute of Technology. This article is based on his plenary talk presented at the CLEO/IQEC Meeting in 1984.

However, it is apparently very difficult to make inductive elements on silicon wafers with present techniques. Even Nature, in her DNA copying machine, dissipates about 100 kT per bit copied. Being, at present, so very far from this $kT\,ln(2)$ figure, it seems ridiculous to argue that even this is too high and the minimum is really essentially zero. But, we are going to be even more ridiculous later and consider bits written on one atom instead of the present 10^{11} atoms. Such nonsense is very entertaining to professors like me. I hope you will find it interesting and entertaining also.

What Bennett pointed out was that this former limit was wrong because it is not necessary to use irreversible primitives. Calculations can be done with reversible machines containing only reversible primitives. If this is done the minimum free energy required is independent of the complexity or number of logical steps in the calculation. If anything, it is kT per bit of the output answer.

But even this, which might be considered the free energy needed to clear the computer for further use, might also be considered as part of what you are going to do with the answer—the information in the result if you transmit it to another point. This is a limit only achieved ideally if you compute with a reversible computer at infinitesimal speed.

Computation with a reversible machine

We will now describe three reversible primitives that could be used to make a universal machine (Toffoli[4]). The first is the NOT which evidently loses no information, and is reversible, being reversed by acting again with NOT. Because the conventional symbol is not symmetrical we shall use an X on the wire instead (see Fig. 3a).

Next is what we shall call the CONTROLLED NOT (see Fig. 3b). There are two entering lines, a and b and two exiting lines, a' and b'. The a' is always the same as a, which is the control line. If the control is activated $a = 1$ then the out b' is the NOT of b. Otherwise b is unchanged, $b' = b$. The table of values

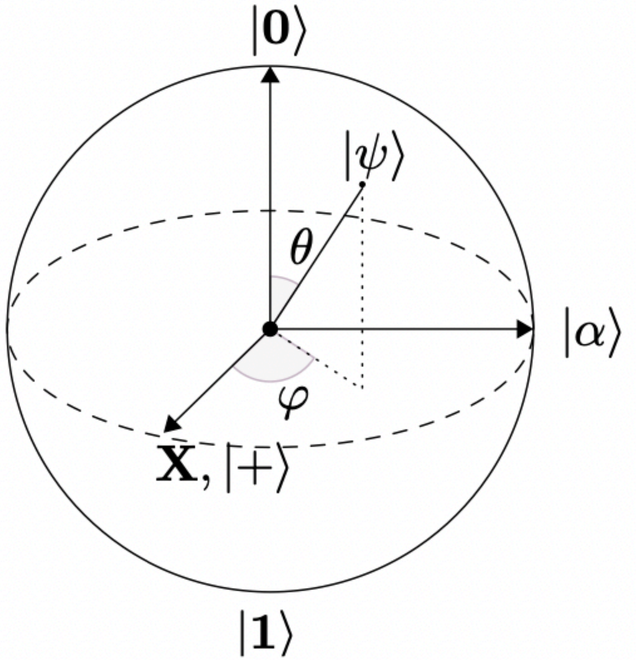
Quantum computing : Approaches

- **Digital quantum computing**: Use qubits to perform computations. There are three steps in general: 1) Initial state-preparation, 2) Implementing unitary evolution using quantum gates, 3) Measurements.

- **Analog quantum computing (continuous/bosonic)**: Use of continuous variables (local Hilbert space is strictly infinite-dimensional like say harmonic oscillator) to carry out state preparation, time evolution, and measurements

States

- Qubits: $d = 2, \quad |0\rangle, |1\rangle$
- Qudits: $d > 2, \quad \text{say} \quad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle + \gamma|2\rangle$
- Qumodes: $d = \infty \quad \text{QHO}$



Quantum gates

$$-\boxed{H}- = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}, \quad |0\rangle -\boxed{H}- |+\rangle$$

$$|+\rangle = \frac{1}{\sqrt{2}}\Big(|0\rangle + |1\rangle\Big)$$

$$-\boxed{X}- = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, \quad |0\rangle -\boxed{X}- |1\rangle$$

$$-\boxed{Z}- = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}, \quad |1\rangle -\boxed{Z}- -|1\rangle$$

$$-\boxed{Y}- = \begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$$

$$-\boxed{P}- = \begin{bmatrix}1 & 0\\ 0 & e^{i\phi}\end{bmatrix}$$

$$-\boxed{R_z(\theta)}- = \begin{bmatrix}e^{-i\theta/2} & 0\\ 0 & e^{i\theta/2}\end{bmatrix}$$

$$-\boxed{S}- = \begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$$

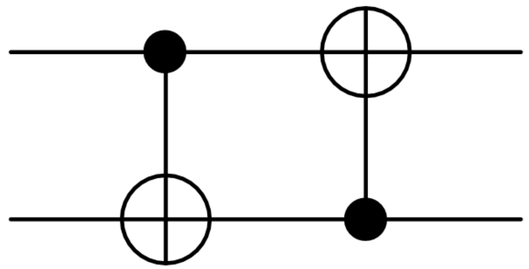
$$-\boxed{T}- = \begin{bmatrix}1 & 0\\ 0 & e^{\frac{i\pi}{4}}\end{bmatrix} = e^{\frac{i\pi}{8}}\begin{bmatrix}e^{\frac{-i\pi}{8}} & 0\\ 0 & e^{\frac{i\pi}{8}}\end{bmatrix}$$

Classical vs. Quantum

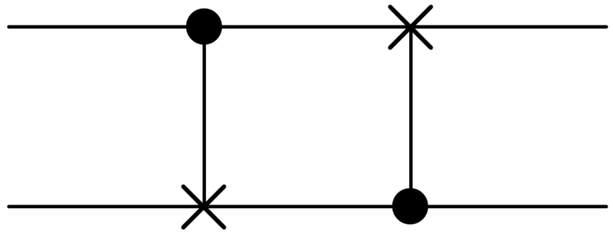
A	B	AND $(A \cdot B)$	OR $(A + B)$	XOR$(A \oplus B)$
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

$\|A\rangle$	$\|B\rangle$	$\|A\rangle$	$\|A \oplus B\rangle$
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

CNOT gate.



Modern notation of CNOT



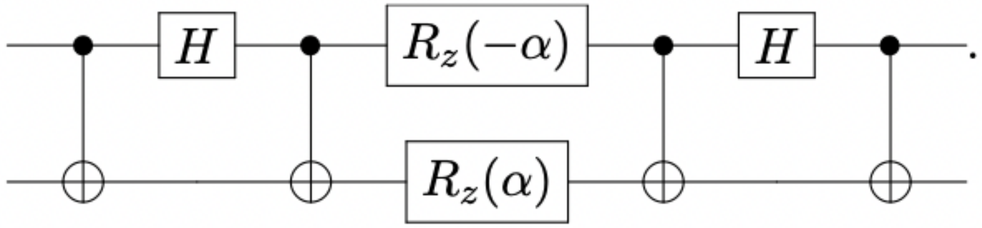
Old notation used by Feynman

What do we want to do?

- One of the problems where theoretical physicists would like to apply these techniques is to understand the time-evolution of some complicated quantum many-body system. Suppose, we have spin-1/2 particle each on two sites with some H below, we would need two qubits to initialise the state say, $|00\rangle$. Now suppose the 4x4 Hamiltonian of this two-site model is given by:

$$H = (X \otimes X) + (Y \otimes Y)$$

We want to do time evolution of this system i.e., $\exp(-iHt)$. We have to represent this unitary operator with quantum (unitary) gates. Note that we have to keep dt sufficiently small, so we have to repeat the circuit below N times where $N = t/dt$. As we can see, we need about $8N$ unitary gates for this simple Hamiltonian and two sites! This cost is very important for practical computations.



QC with continuous variables

- For fermionic systems, like Ising model, the qubit approach is generally preferred but for models with bosonic degrees of freedom (where the local Hilbert space dimension is infinite), the more natural setting is one of oscillator (qumodes). Suppose, we consider the famous Bose-Hubbard model where the H is given by:

$$H = J\sum_{\langle ij\rangle} a_i^\dagger a_j + \frac{1}{2}U\sum_i \hat{n}_i(\hat{n}_i - 1)$$

where we have used create /annihilation operators and the number operators. The first term denotes the hopping of bosons between neighbouring sites and second term is the on-site potential term.

We can write the time-evolution operator as:

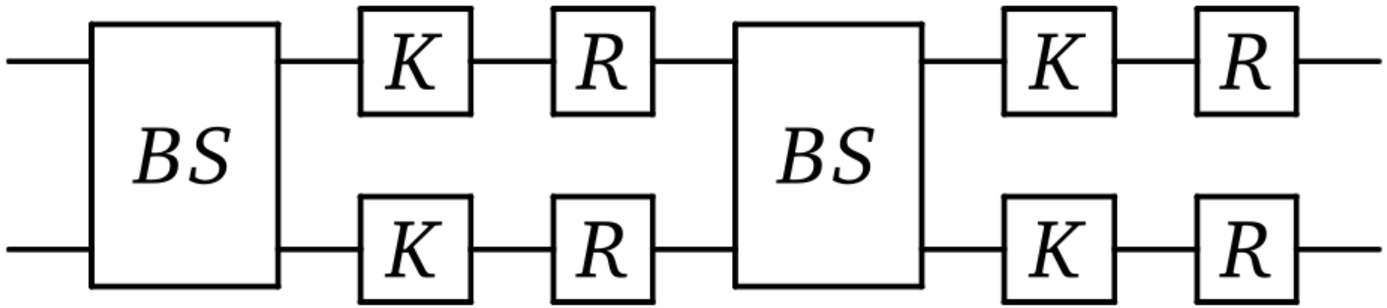
$$e^{iHt} = \left[BS\left(\theta,\phi\right)\left(K(r)R(-r)\otimes K(r)R(-r)\right)\right]^N + \mathcal{O}\left(t^2/N\right) \quad ;$$

$$\theta = -Jt/N, \phi = \pi/2,\ r = -Ut/2N$$

where BS is the beam-splitter gate, K is the Kerr gate, and R is the rotation gate. These gates are qumodes equivalent of the gates we saw before. For example, $K(\kappa) = \exp(i\kappa\hat{n}^2)$.

QC with continuous variables

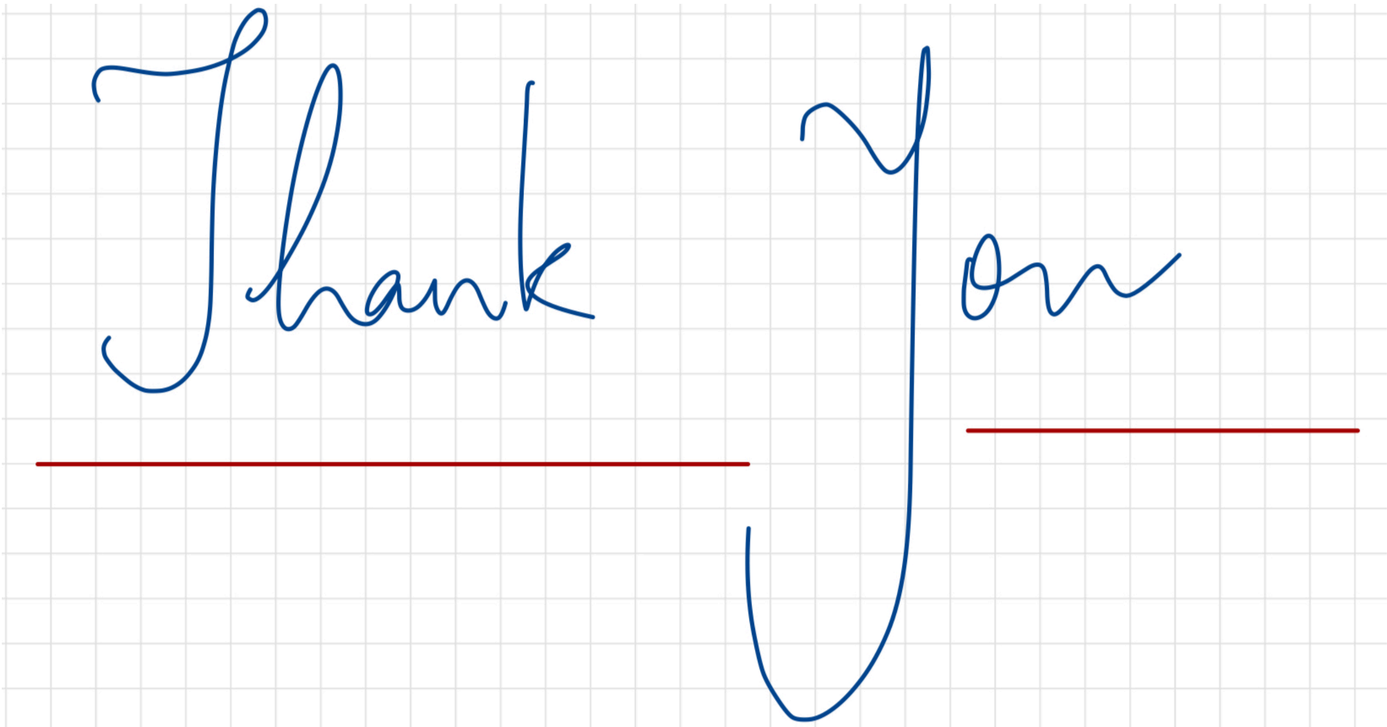
- One step of time evolution (remember we need to do N steps) is shown below:

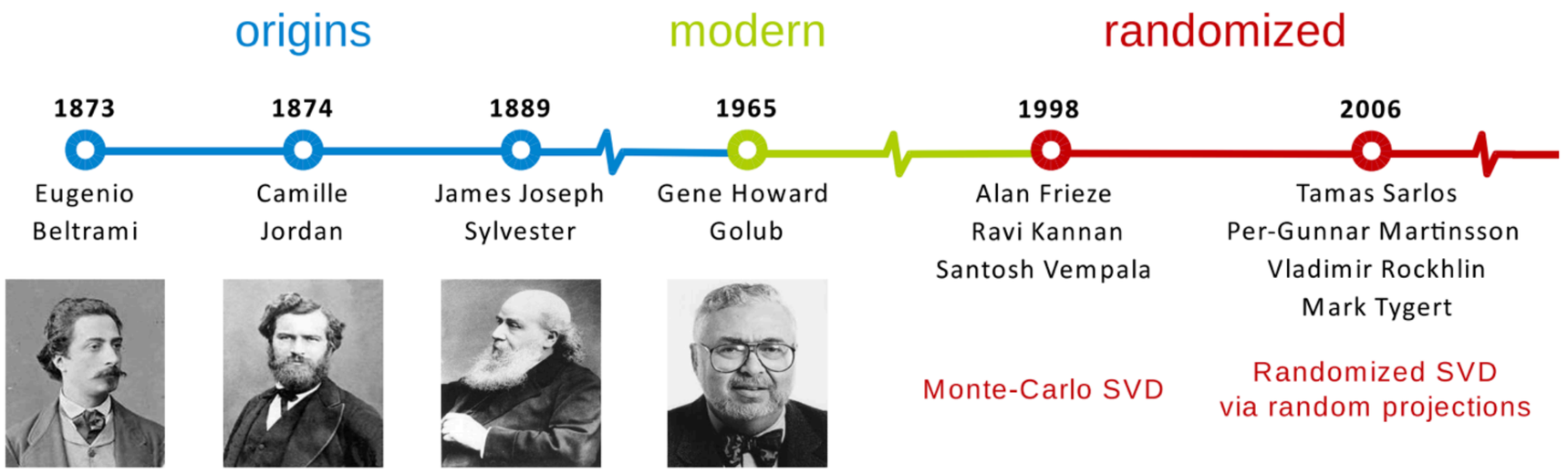


Summary

Tensor network methods have potential to assist in various interesting problems in Physics. On one hand, it can efficiently reproduce the ground state of several quantum systems with MPS while on the other hand it can also describe real-space RG in various dimensions and can help us in understanding spin models, complex action systems, gauge theories etc. It is indeed a very exciting approach to numerical aspect of RG!

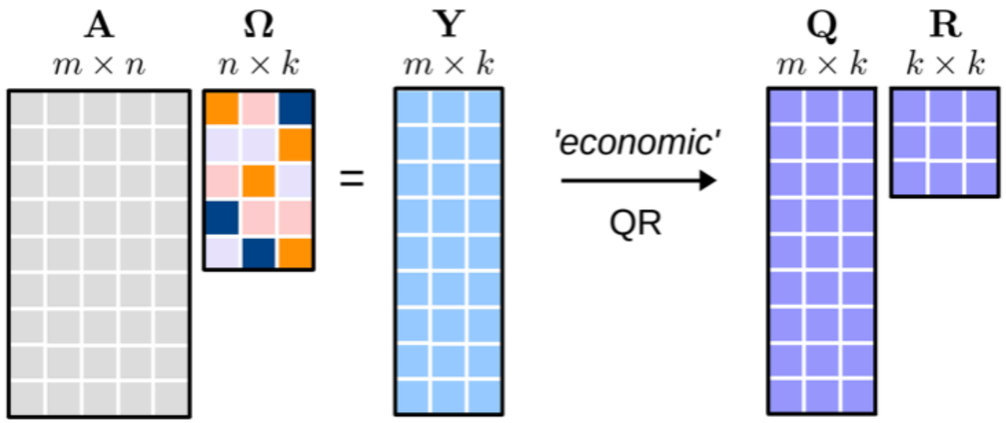
Looking quantum mechanically, these models and several others can be (hopefully) studied in future using qubits and qumodes much more efficiently. This would tell us much more about the time evolution of quantum systems which is a hard problem.

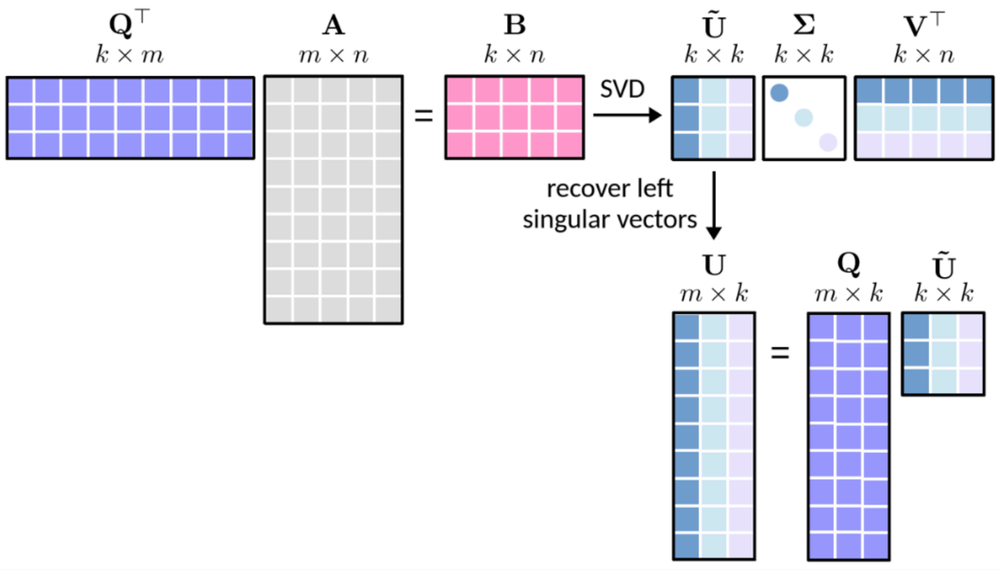

Thank You



arXiv:1608.02148

Randomised SVD





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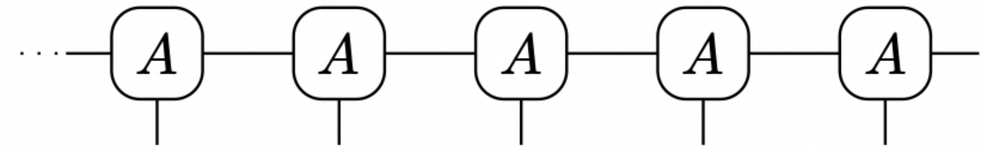
$$|\Psi(A)\rangle =$$ 

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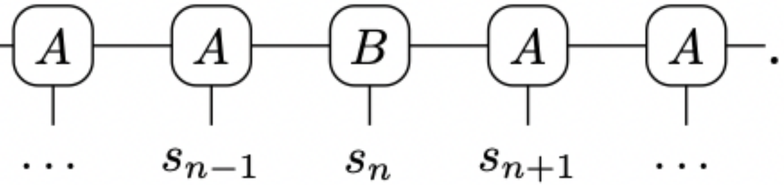
$$d^N \to Nd\chi^2$$

$$|\Phi_k(B)\rangle = \sum_n \mathrm{e}^{ikn}$$ 

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Van Damme et al. 1907.02474

CV gates

Displacement $D_i(\alpha) = e^{(\alpha \hat{a}_i^\dagger - \alpha^* \hat{a}_i)}$

Rotation $R_i(\phi) = e^{i\phi \hat{n}_i}$

Squeezing $S_i(z) = e^{\frac{1}{2}(z^* \hat{a}_i^2 - z \hat{a}_i^{\dagger 2})}$

Beam Splitter $BS_{i,j}(\theta, \phi) = e^{\theta(e^{i\phi} a_i^\dagger a_j - e^{-i\phi} a_i a_j^\dagger)}$

Cubic Phase $V_i(\gamma) = e^{\frac{i\gamma}{6} \hat{x}_i^3}$

Matrix rep. of these gates.

$$D_i(\alpha) = \begin{pmatrix} b_i \\ b_i^\dagger \\ \mathbb{I} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \alpha \\ 0 & 1 & \alpha^* \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_i \\ a_i^\dagger \\ \mathbb{I} \end{pmatrix}$$

$$R_i(\phi) = \begin{pmatrix} b_i \\ b_i^\dagger \end{pmatrix} = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix} \begin{pmatrix} a_j \\ a_j^\dagger \end{pmatrix}$$

$$S_i(re^{i\theta}) = \begin{pmatrix} b_i \\ b_i^\dagger \end{pmatrix} = \begin{pmatrix} \cosh r & -e^{-i\theta} \sinh r \\ -e^{-i\theta} \sinh r & \cosh r \end{pmatrix} \begin{pmatrix} a_j \\ a_j^\dagger \end{pmatrix}$$

Now suppose that similar to example with qubits where we had

$$H = \sigma_z \otimes \sigma_z + \sigma_x \otimes \sigma_x + \Pi_2$$

could have been represented $e^{-i\hat{H}t}$ by quantum circuit.

Suppose $$\hat{q}_k = \frac{\hat{a}_k + \hat{a}_k^\dagger}{\sqrt{2}}$$

$$\hat{p}_k = \frac{\hat{a}_k - \hat{a}_k^\dagger}{i\sqrt{2}}$$

are the quadrature operators for mode 'k' satisfying $[\hat{q}_k, \hat{p}_l] = i\delta_{kl}$

Then

Displacement $\longrightarrow e^{i\hat{q}_k s_1}$ $\quad s_i \in \mathbb{R}$ for all 'i'

Squeezing $\longrightarrow e^{i(\hat{q}_k\hat{p}_k + \hat{p}_k\hat{q}_k)s_2}$

Beam-Splitter $\longrightarrow e^{i(\hat{p}_k\hat{q}_l - \hat{q}_k\hat{p}_l)}$

$\longrightarrow$ quadratic Hamiltonian