Probing Holographic Dualities with lattice supersymmetric YM theories.

MIT Lattice Colloquium 2021

1709.07025, 2003.01298, 2010.00026, 2104.xxxxx

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Talk

1 of 1

February 25, 2021

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What this talk could have been

- ★ Introduction to tensor renormalization group (TRG) approach to efficiently solve spin models (Ising, O(2), O(3) etc.) and lattice gauge theories: 2d/3d $U(1)$ and $SU(2)$ Wilson action + fundamental matter.

- ★ This `has' the potential to substitute the Monte-Carlo approach for wide range of 2d/3d/4d models since there is no `sign problem'. However, not a panacea! For ex: 2d $U(1)$with θ-term was studied few years back in 1911.06480. This has exact solution at strong coupling but numerical studies essential as $\beta \gg 1$. A possible window to understanding topology & other rich properties in lattice field theories. Also studies of 2d O(2) & $3d$ $O(2)$ at finite chemical potential!

Some work I've done — 1901.11443, 2004.06314 + some upcoming!

What this talk will be

- Introduction to how we can discretize and study certain supersymmetric gauge theories on the lattice.

- What opportunities open up for the lattice at strong couplings and large N ('t Hooft limit) in various dimensions and how it can sharpen and refine our understanding of holography and string theory. Some future prospects on how we can continue this program and computational advances we need to achieve to make things much more interesting!

Outline

- ★ Holography for 0+1-dimensional matrix models and generic SYM theories in $4 \geq d > 1$ [for ex: $\mathcal{N} = 4$ SYM] on the lattice.
- ★ Phase structure of a supersymmetric matrix model at finite couplings and large N at finite temperatures.
- ★ $2d$ & $3d$ SYM on the lattice and thermodynamics of the dual black branes
- ★ Future directions!

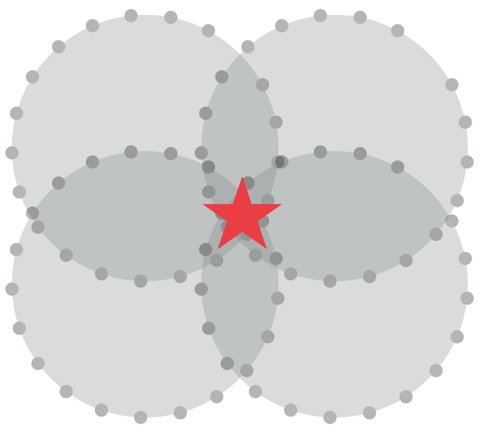
Holography

The idea that a quantum-gravitational theory in one higher dimension ($d + 1$) dimensions is related to some quantum field theory (without gravity) in one lower dimension (d) on its boundary.

It is now widely believed that the any consistent theory of quantum gravity will admit a holographic description.

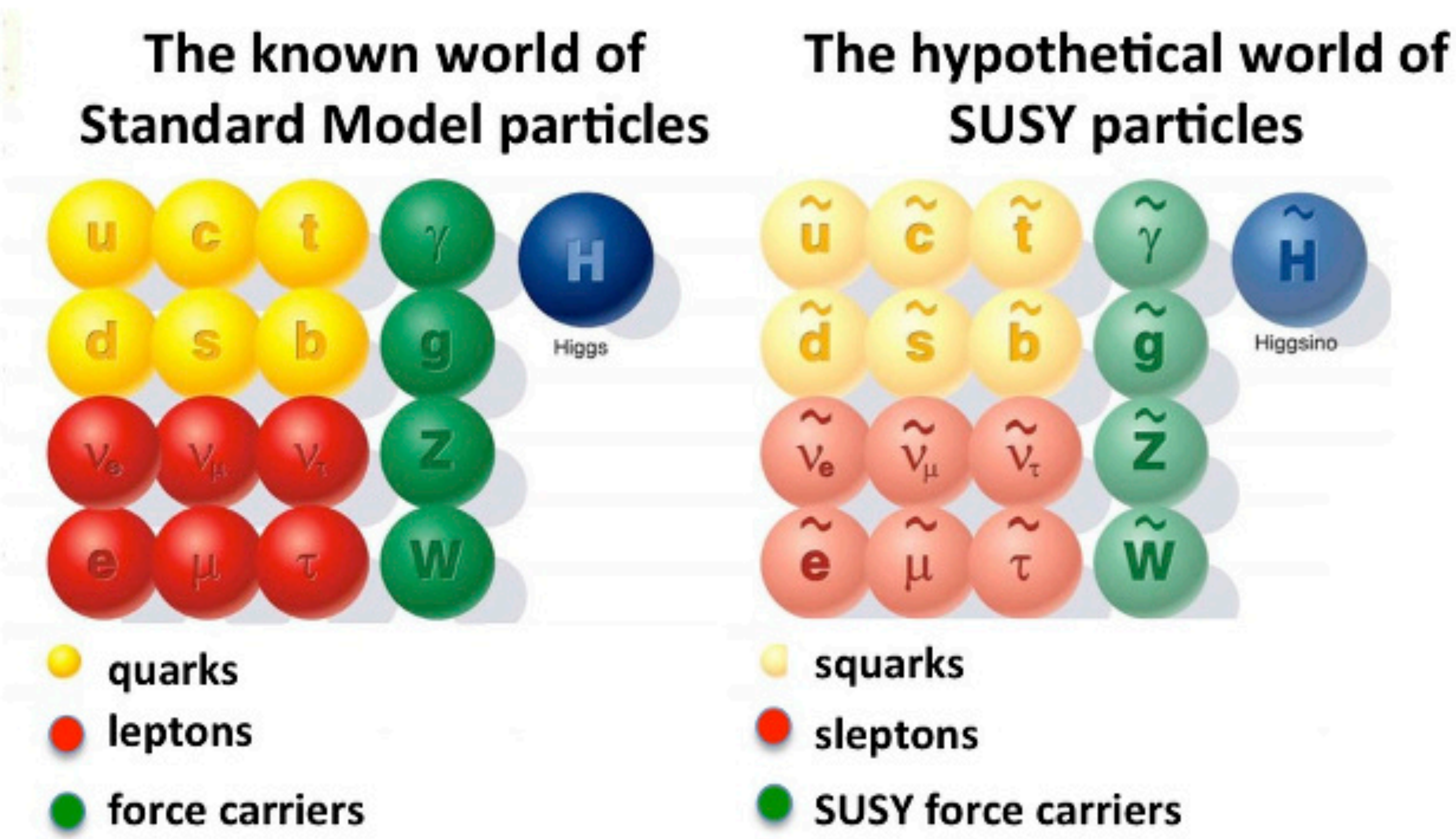
First hints came in 1970s, when Stephen Hawking and Jacob Bekenstein found that the black hole entropy was proportional to the area of its event horizon.

$$S_{BH} = \frac{k_B c^3 A}{4G\hbar}$$



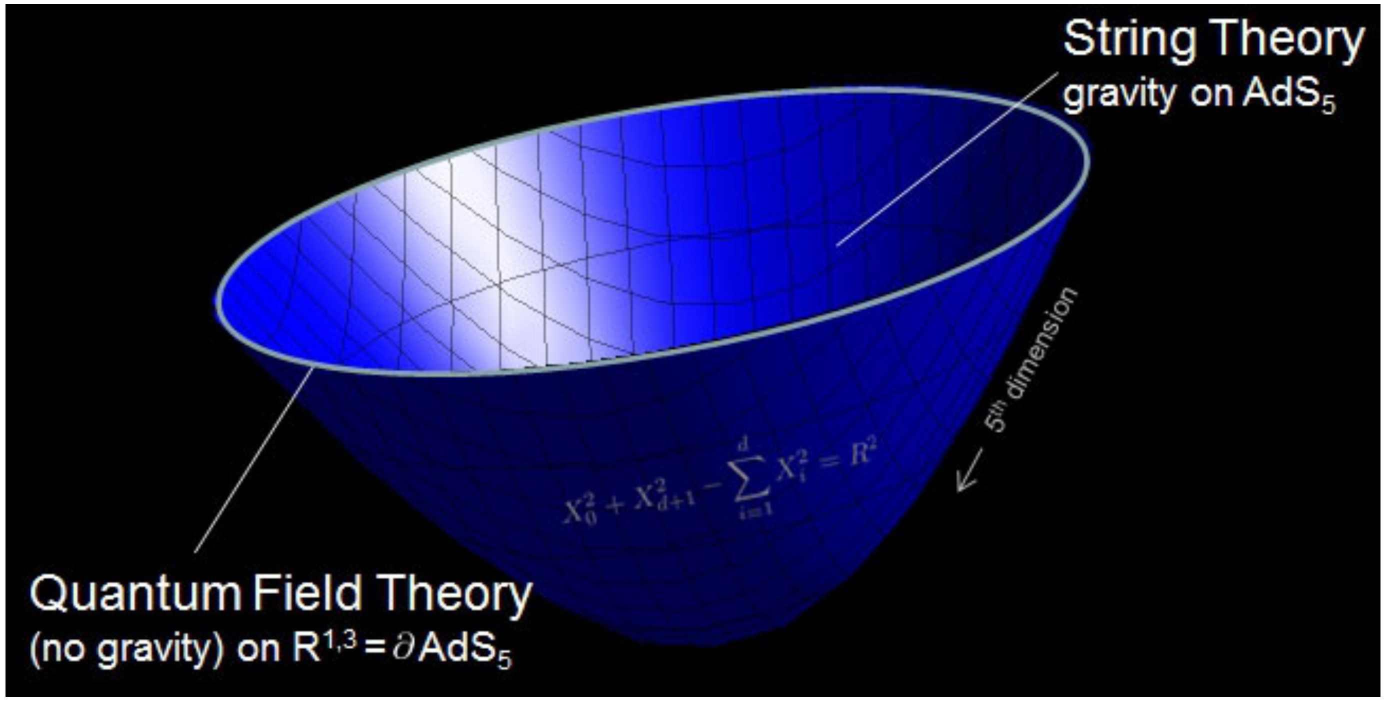
Supersymmetry!

It is elegant, beautiful, broken, and has not been experimentally observed.



AdS/CFT [Maldacena, 1997]

A well-defined correspondence was conjectured between a five-dimensional quantum theory of gravity in Anti- de Sitter (AdS) space-time and four-dimensional super-conformal field theory (CFT) on the boundary. In the limit of $N \to \infty, \lambda \gg 1$, the quantum gravity reduces to Einstein-like gravity in the bulk.



But not only AdS_5/CFT_4!

But, there is nothing special about the fundamental idea of holography to the pair of 4 and 5 dimensions. Within one year, it was rigorously defined for maximal supersymmetric gauge theories for $d < 4$ even though they are no conformal. One of the most studied pair is AdS_3/CFT_2

Maximally supersymmetric Yang-Mills theory in $p+1$-dimensions is dual to Dp-branes in supergravity at low temperatures in a special limit (large N, strong coupling). In other words, the supergravity solutions corresponding to $p+1$ superYang-Mills are black p-brane (Dp-brane) solutions.

higher dimensional black hole version in ST.

Zoo of SYM theories!

The starting point is the action of ten-dimensional $\mathcal{N}=1$ SYM theory:

$$S = \frac{1}{g^2}\int d^{10}x \ \mathrm{Tr}\ \left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}D_\mu\gamma^\mu\psi\right)$$

where D is the covariant derivative, ψ is a spinor with 16 components (real).

16 supercharges:

D=10, $\mathcal{N}=1$ $\rightarrow$ D=6, $\mathcal{N}=2$ $\rightarrow$ D=4, $\mathcal{N}=4$ $\rightarrow$ D=3, $\mathcal{N}=8$ $\rightarrow$ D=2, $\mathcal{N}=(8,8)$

8 supercharges:

D=6, $\mathcal{N}=1$ $\rightarrow$ D=4, $\mathcal{N}=2$ $\rightarrow$ D=3, $\mathcal{N}=4$

4 supercharges:

D=4, $\mathcal{N}=1$ $\rightarrow$ D=3, $\mathcal{N}=2$

$\mathcal{N} = 4$ super-Yang-Mills (SYM)

Obtained by dimensionally reducing the ten-dimensional SYM theory down to four dimensions. It is a conformal field theory, beta-function vanishes, consists of six scalars, sixteen real fermions, all massless and in the adjoint representation of the $SU(N)$ gauge group. Simplest interacting QFT in four dimensions.

The action consists of kinetic, Yukawa, quartic scalar commutator terms and are all related by supersymmetry. The superconformal algebra include $SU(4) = \text{Spin}(6)$ symmetry and is part of R-symmetry group apart from the usual $SO(4)$ Euclidean group. At finite temperatures, SUSY is broken. Sometimes dubbed as close cousin of QCD (not physical though!)

Supersymmetry (SUSY) on the lattice

Beset by difficulties from the start because of SUSY algebra. The algebra is an extension of Poincare algebra by supercharges Q and . Roughly, $\{Q, \overline{Q}\} \sim P_\mu$ and P_μ generates infinitesimal translations which is broken on the lattice. SUSY algebra not satisfied at the classical level.

Alternative:

Preserve a subset of this algebra and check (expect!) that the supersymmetry is restored as continuum limit is taken. This idea has led to an improved understanding and has used for the results mentioned later in this talk. For review see: **0903.4881**

[Cohen, Kaplan, Katz, Unsal, Catterall, Sugino]
during 2000-2008 using different but equivalent approaches.

Still some not possible

One of the requirements that twisting procedure can be done is that one must start with sufficient SUSY in the continuum theory (2^d). Clearly maximal SUSY theories in $1 \leq d \leq 4$ satisfy this easily. For direct head-on dealing with fine-tuning and studies of $\mathcal{N} = 1$ SYM in four dimensions, see work by [Bergner, Münster, Montvay et al.]. See an earlier talk in this colloquium series for more on this.

Theory	**R-symmetry group**	**Orbifolding**	**Maximal Twist**
$d = 2, \mathcal{Q} = 4, \mathcal{N} = 2$	$SO(2) \bigotimes U(1)$	Yes	Yes
$d = 2, \mathcal{Q} = 8, \mathcal{N} = 4$	$SO(4) \bigotimes SU(2)$	Yes	Yes
$d = 2, \mathcal{Q} = 16, \mathcal{N} = 8$	$SO(8)$	Yes	Yes
$d = 3, \mathcal{Q} = 4, \mathcal{N} = 1$	$U(1)$	No	No
$d = 3, \mathcal{Q} = 8, \mathcal{N} = 2$	$SO(3) \bigotimes SU(2)$	Yes	Yes
$d = 3, \mathcal{Q} = 16, \mathcal{N} = 4$	$SO(7)$	Yes	Yes
$d = 4, \mathcal{Q} = 4, \mathcal{N} = 1$	$U(1)$	No	No
$d = 4, \mathcal{Q} = 8, \mathcal{N} = 2$	$SO(2) \bigotimes SU(2)$	No	No
$d = 4, \mathcal{Q} = 16, \mathcal{N} = 4$	$SO(6)$	Yes	Yes

Lattice $\mathcal{N} = 4$ SYM

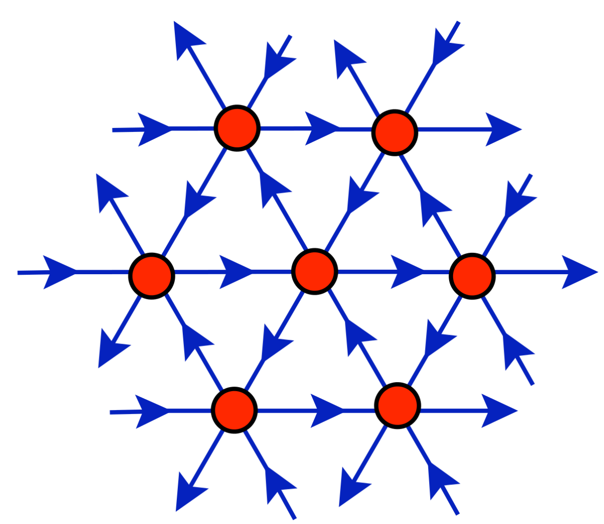
This talk will present results based on the geometric construction and idea of topologically twisting (maximal twist) a supersymmetric gauge theory. This generates the 0-form supercharges needed to preserve a subset of SUSY algebra. In some sense, this is just a way of rewriting original fields and is justified for flat Euclidean space. Supercharges are broken into p -forms and then put on the lattice sites, links, plaquettes respectively.

Basic idea: Take maximum subgroup $SO(4) \subset SO(6)$ of the R-symmetry group and construct $SO(4)_{\text{tw.}} = \text{diag}[\text{SO}(4)_{\text{E}} \times \text{SO}(4)_{\text{R}}]$. This gives a nilpotent (i.e. $Q^2 = 0$) supercharge which can then be preserved exactly on the lattice. If we look at the table, now it is clear why certain theories cannot be twisted in this sense!

Special lattice required

- To reduce the fine-tuning to minimum possible and to identify the twisted fields in a consistent manner, we cannot just work with hypercubic lattice. We need what is called A_4^* lattice. Four-dimensional version of the triangular lattice shown below.

- S_5 point group symmetry and five links which is natural setting to lay out ten field components of the $\mathcal{N} = 4$ SYM theory. But, basis vectors are not orthogonal.



Public Code

SUSY lattice community is small and large-scale lattice calculations still in early days. We make our parallel, four-dimensional, arbitrary N code available on GitHub. Updated version release to follow on Computer Physics Communications (CPC) soon.

so that the full improved action becomes

$$
\begin{aligned}
S_{imp} &= S'_{exact} + S_{closed} + S'_{soft} \qquad (3.10)\\
S'_{exact} &= \frac{N}{2\lambda_{\text{lat}}} \sum_n \text{Tr}\Bigg[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) - \chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{b]}(n) - \eta(n)\overline{\mathcal{D}}^{(-)}_a\psi_a(n) \\
&\qquad + \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a(n) + G\sum_{a\neq b}\left(\det\mathcal{P}_{ab}(n) - 1\right)\mathbb{I}_N\right)^2\Bigg] - S_{det} \\
S_{det} &= \frac{N}{2\lambda_{\text{lat}}} G \sum_n \text{Tr}\left[\eta(n)\right] \sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n)\right] \text{Tr}\left[\mathcal{U}_b^{-1}(n)\psi_b(n) + \mathcal{U}_a^{-1}(n+\widehat{b})\psi_a(n+\widehat{b})\right] \\
S_{closed} &= -\frac{N}{8\lambda_{\text{lat}}} \sum_n \text{Tr}\left[\epsilon_{abcde}\ \chi_{de}(n+\widehat{\mu}_a+\widehat{\mu}_b+\widehat{\mu}_c)\overline{\mathcal{D}}^{(-)}_c\chi_{ab}(n)\right], \\
S'_{soft} &= \frac{N}{2\lambda_{\text{lat}}}\mu^2\sum_n\sum_a\left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a(n)\overline{\mathcal{U}}_a(n)\right] - 1\right)^2
\end{aligned}
$$

The lattice action is obviously very complicated

(For experts: $\gtrsim$100 inter-node data transfers in the fermion operator)

To reduce barriers to entry our parallel code is publicly developed at
github.com/daschaich/susy

Evolved from MILC lattice QCD code, presented in arXiv:1410.6971

Lower-dimensional SYM theories

As nice as it sounds, on the lattice $\mathcal{N} = 4$ SYM is extremely hard and probably not possible to study at large N and λ in practice using classical computers. Also, there is a sign problem which we have observed for $\lambda \geq 5$. Additional complications because of being a super conformal field theory with no scales, moduli etc.

My research has focused over the years on the lower-dimensional version of this theory which are non-conformal and the computational costs are under more control. Also, sign problem does not seem to play a role for range of couplings for interesting finite-temperature black hole Physics. Note that they are also sometimes referred to as `non-extremal' black holes.

Results presented in this talk

★ 0+1-dimensional matrix QM (BFSS and BMN models) [In progress, 2021]

★ 1+1-dimensional $\mathcal{N}$ = (8,8) SYM at finite temperatures [published 2017]

★ 2+1-dimensional SYM [published 2020]

Matrix Models

Obtained by dimensional reduction of $\mathcal{N} = 1$ SYM from ten dimensions down to one. The theory has SU(N) gauge symmetry and SO(9) internal symmetry group corresponding to nine scalars. Simplest holographic gauge theory with well-defined gravity dual.

$$S_{\rm BFSS} = \frac{N}{4\lambda}\int dt \mathrm{Tr}\Big[(D_t X^i)^2 - \frac{1}{2}\left[X^i, X^j\right]^2 + \Psi^T D_t \Psi + i\Psi^T\gamma^j[\Psi, X^j]\Big]$$

$$S_{\rm BMN} = S_{\rm BFSS} + S(\mu)$$

μ-terms break the $SO(9) \to SO(6) \otimes SO(3)$. BFSS has a single deconfined phase but BMN model admits a *deconfinement* phase transition!

BFSS := Banks-Fischler-Shenker-Susskind [1996]
BMN := Berenstein-Maldacena-Nastase [2002]

State-of-the-art lattice results for BFSS

- a_0 is known from supergravity (SUGRA) calculations.
- Finite-T corrections => α' corrections in string theory

The coefficients $a_1, a_2 ..$ etc. are not known from supergravity. We only know that corrections start at $(\alpha')^3 \sim T^{9/5}$

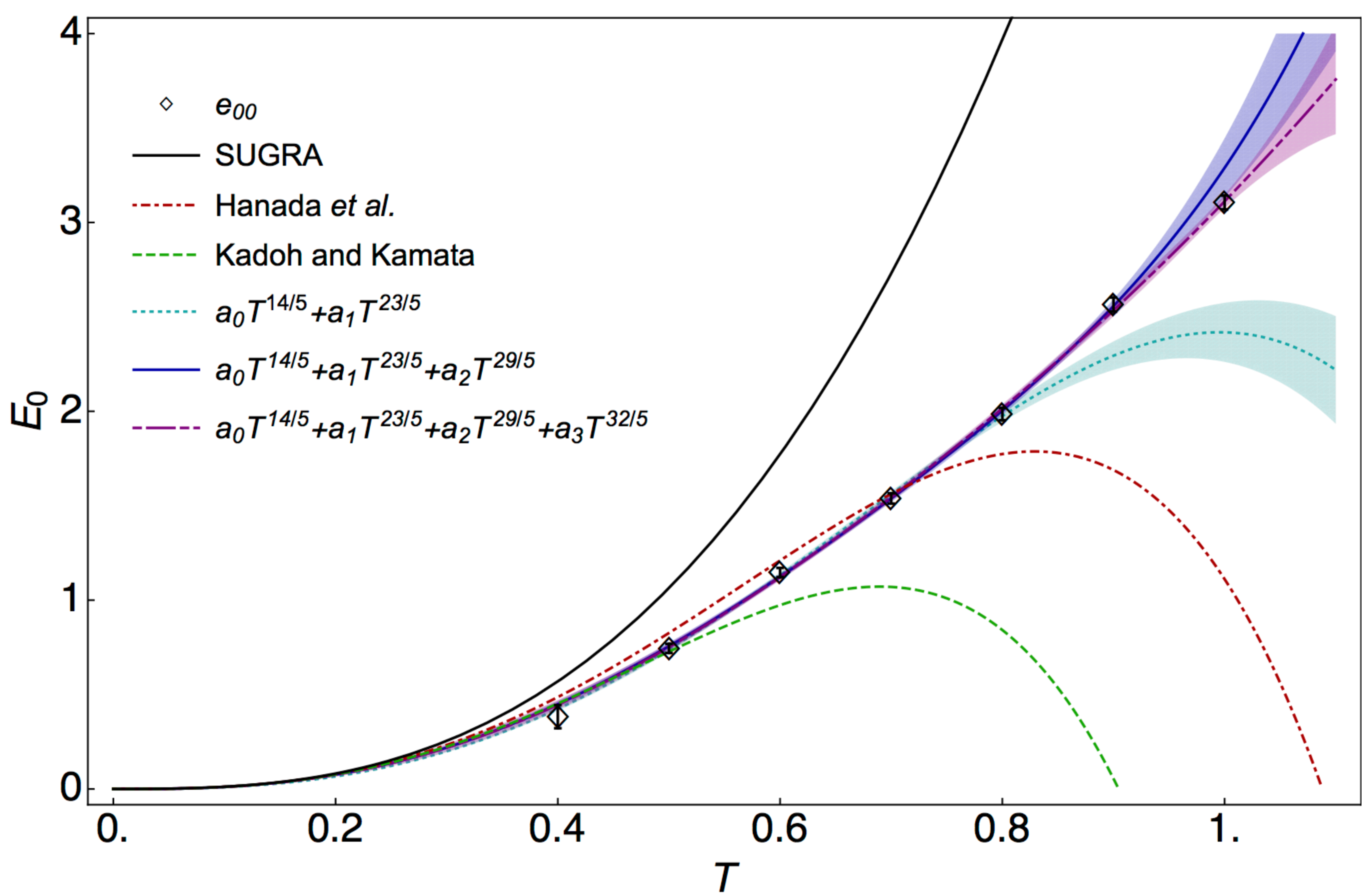


Figure from arXiv: 1606.04951

(Hanada, Ishiki et al.)

How such nice results?

They explored $N = 32$ and took the continuum limit. Taking continuum limit almost trivial since one-dimensional! But nice results are possible because their code parallelizes over the matrix degrees of freedom. Parallelization over N is essential to reproduce holographic behaviour accurately in this model. We have not yet explored making our MILC lattice QCD-based code to do this yet. We only have parallelization over lattice volume, not so useful in 1d!

BMN matrix model

$$S_{BMN} = S_{BFSS} - \frac{N}{4\lambda}\int d\tau \ \mathrm{Tr}\left(\frac{\mu^2}{3^2}\left(X^i\right)^2 + \frac{\mu^2}{6^2}\left(X^M\right)^2 + \frac{2\mu}{3}\epsilon_{IJK}X^iX^jX^k + \frac{\mu}{4}\overline{\Psi}^\alpha\left(\gamma^{123}\right)_{\alpha\beta}\Psi^\beta\right)$$

The flat directions of the BFSS model are lifted by giving masses to $SO(3)$ and $SO(6)$ scalars. In addition, there is a cubic scalar term which is also known as 'Myers term' plus a fermion term. Dual gravity solution applicable when $g = \lambda/\mu^3 \gg 1$ with $\mu \ll 1$, $N \to \infty$.

Results at $g \to 0$ [see most recently O'Connor et al. 1805.05314 + more]

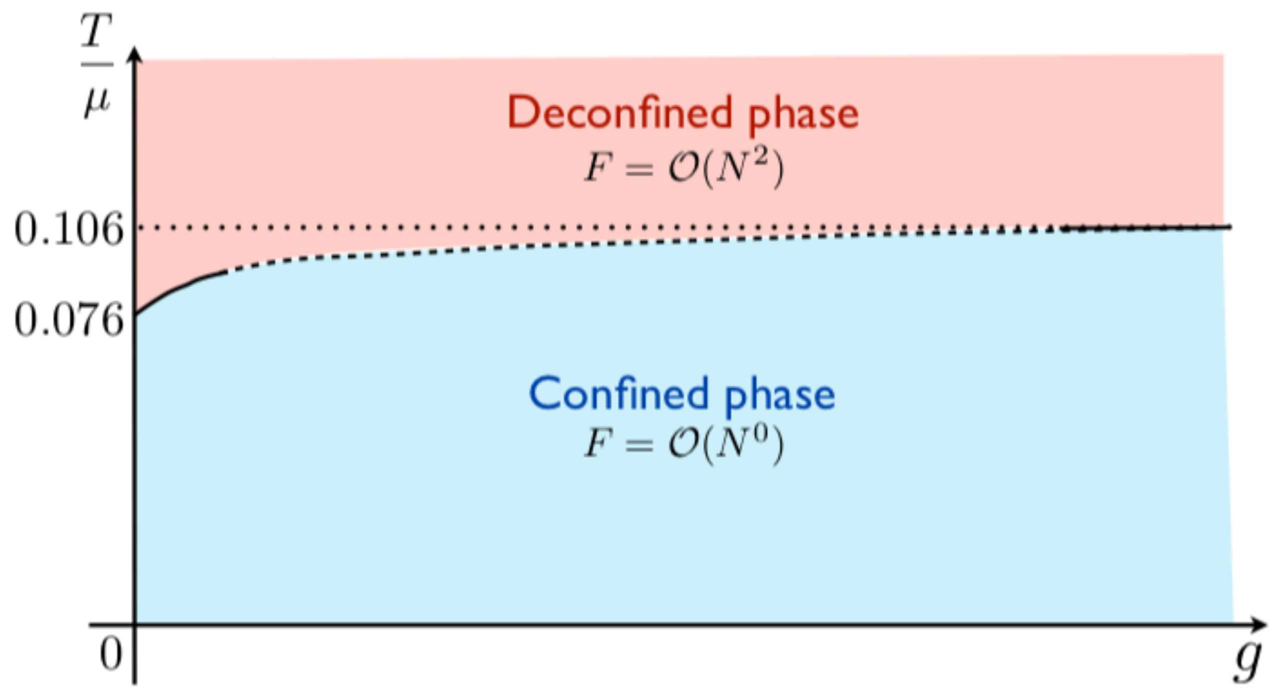
In this limit, the theory can be studied perturbatively. Note that since $g = \dfrac{\lambda}{\mu^3}$, this is the large μ limit. In this case, the model becomes a supersymmetric gauged Gaussian model. It was well-studied and the critical temperature was determined to be:

$$\left.\frac{T}{\mu}\right|_c = \frac{1}{12\ln 3}\left[1 + O(\lambda) + O(\lambda^2)\right]$$

which increases with λ but is bounded by some gravity result as we will see soon. For $\lambda = 0$, we have $(T/\mu)_c \approx 0.076$.

Conjectured phase diagram

For this model, the phase structure should look like one given below. The solid lines are known results from perturbation theory and gravity computations. The dashed lines have no explanation yet. There might also be non-smooth behaviour unlike what is shown.



1411.5541 [Costa, Penedones, Greenspan, Santos]

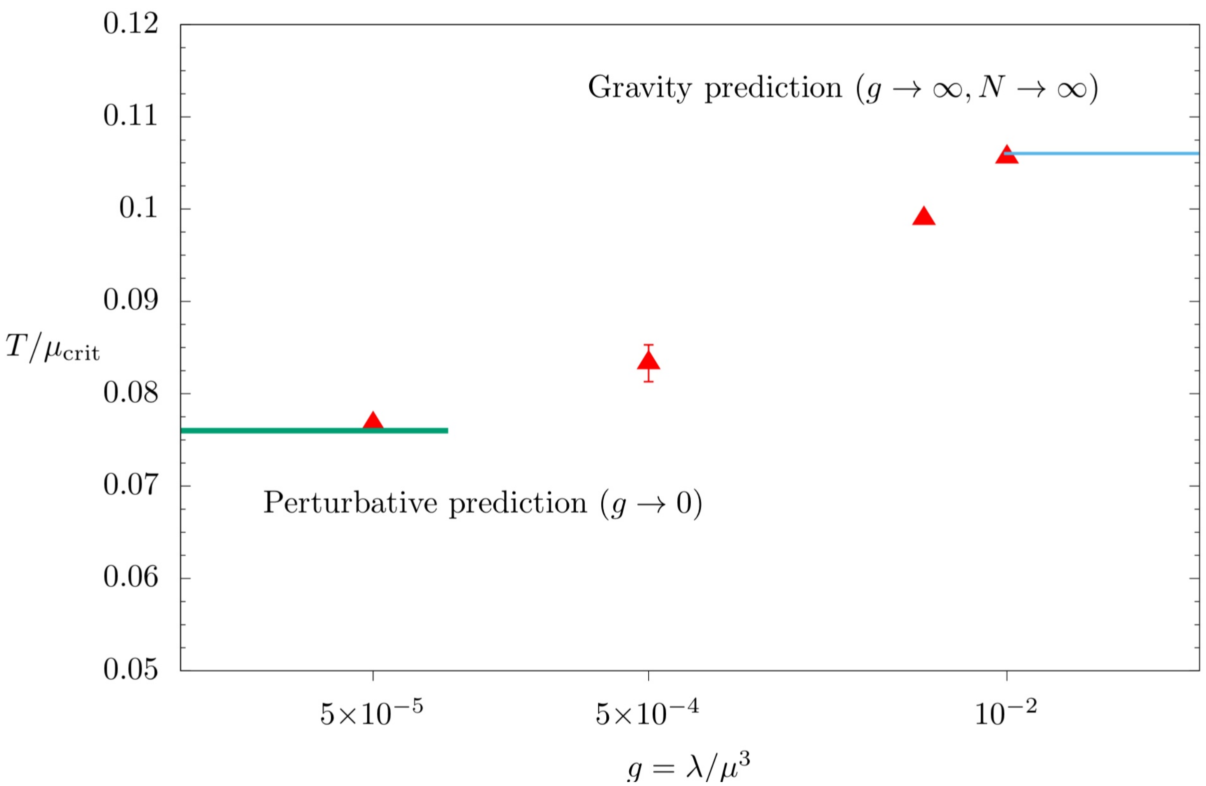
Results from gravity computation i.e. $g \gg 1$

We need to reproduce the results using lattice. The (internal) energy density has a parametric form given by $f(\mu) \sim a_0(\mu)T^{14/5}$ similar to BFSS matrix model. However, now, $a_0(\mu)$ is not known. Lattice should be able to compute this along the same lines as done for BFSS where $a_0(0) \approx 7.41$ [Open]

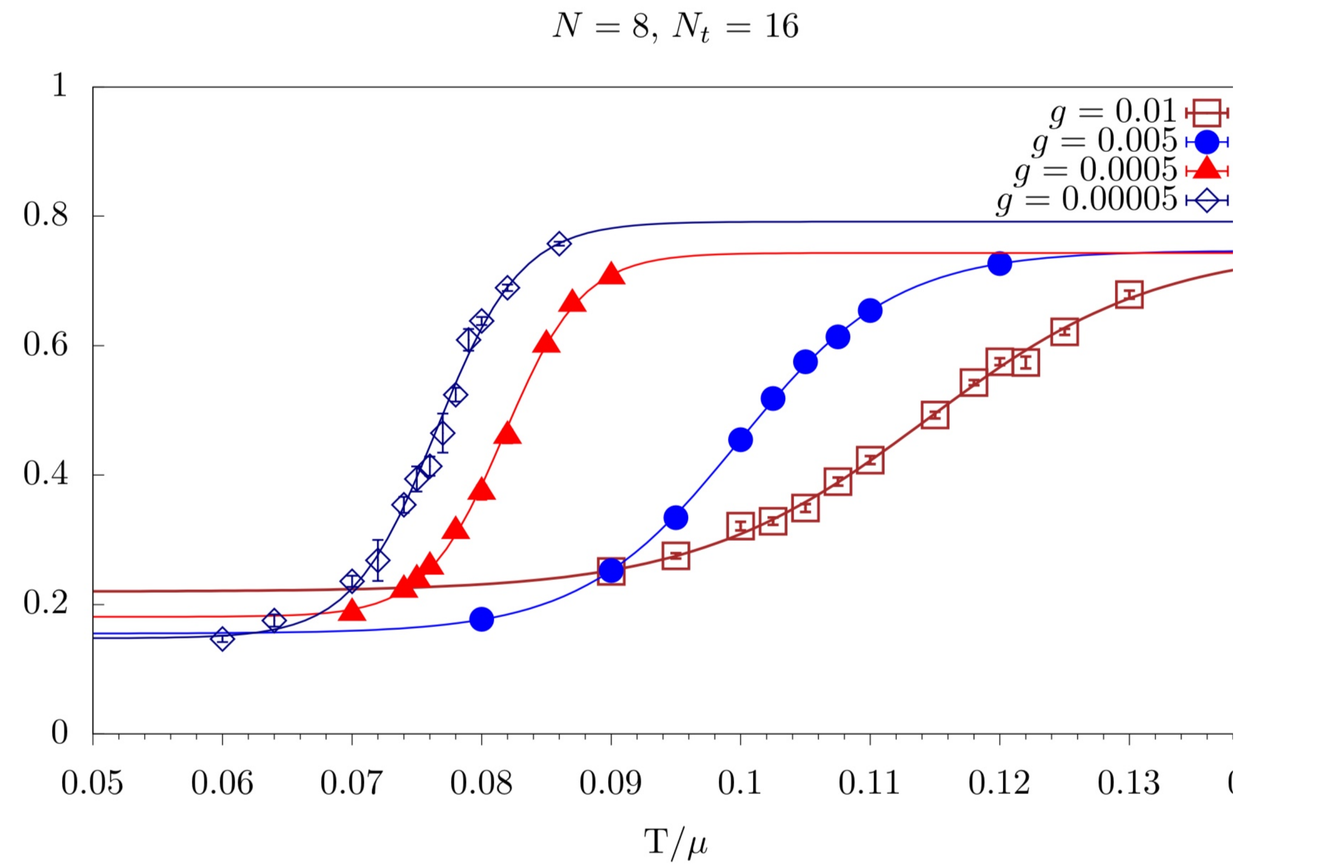
The critical value of $T/\mu \sim 0.106$ was computed by finding the location where the free energy has a zero in 1411.5541. Should be match to this critical value using the lattice. This is work in progress!

Also using the holographic dictionary, the scalar squares i.e. $\mathrm{Tr}(X^2)$ in super YM theories are related to the topology of the black hole horizon in the dual gravity interpretation.

Results - 1 [preliminary] 2003.01298, 2104. XXXXX



Finite coupling phase transition from lattice calculations.



Polyakov loop as order parameter

1+1-dimensional SYM

Dimensionally reduce the four-dimensional theory we have discretized on A_4^* lattice down to two dimensions. Fermion with SUSY breaking boundary conditions.

- Dimensionless couplings, $r_x = \sqrt{\lambda}L$, $r_\beta = \sqrt{\lambda}\beta = 1/t$, and lattice aspect ratio $\alpha = L/\beta$.
- Phase transition between localized black hole and black string conjectured using gravity computations at $\alpha^2 r_\beta \sim 2.45$
- This is a topological transition on the gravity side, dual to deconfinement transition on the gauge theory side. In the large N limit, even at weak coupling, there is a first -order phase transition [argued by some to be like the kind of phase transitions seen in the famous one-plaquette $2d$ pure lattice gauge theory by Gross/Witten/Wadia]. This transition continues to the strong coupling limit.

Regime of validity in coupling space

To have a valid SUGRA description we need to satisfy certain conditions:

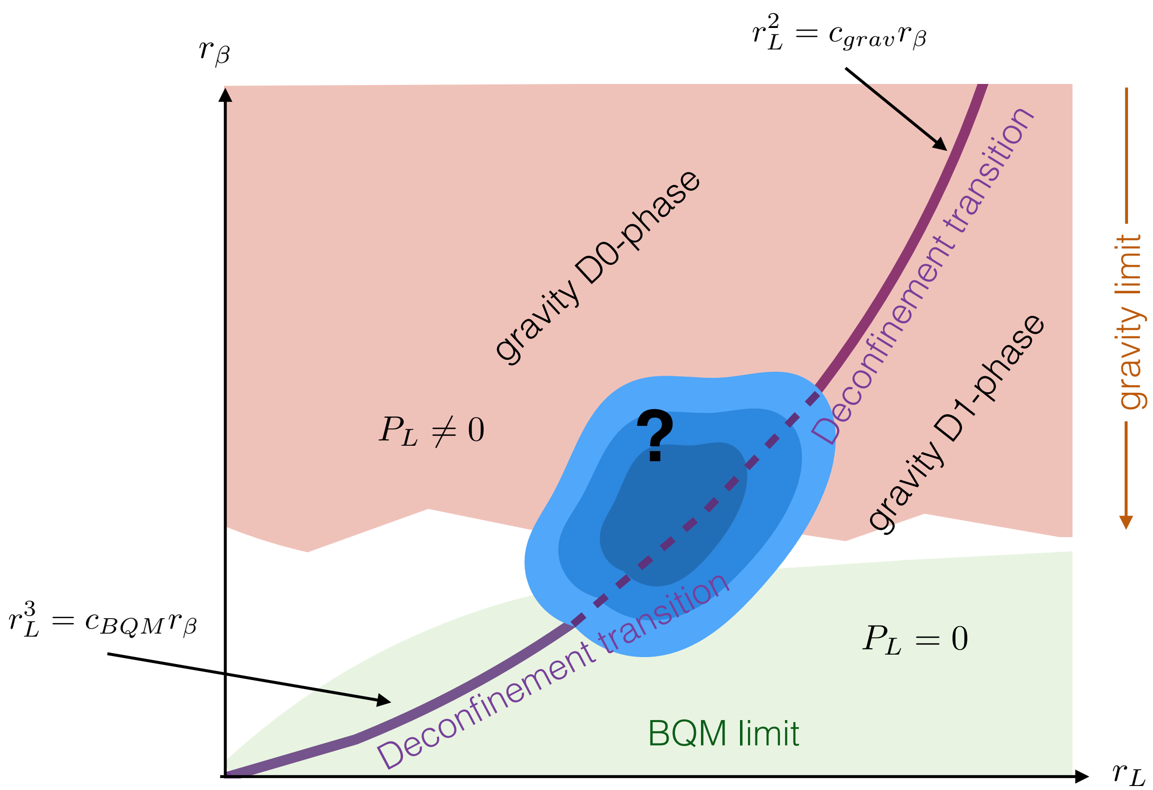
1. Radius of curvature should be large in units of α'. This implies $r_\beta \gg 1$.

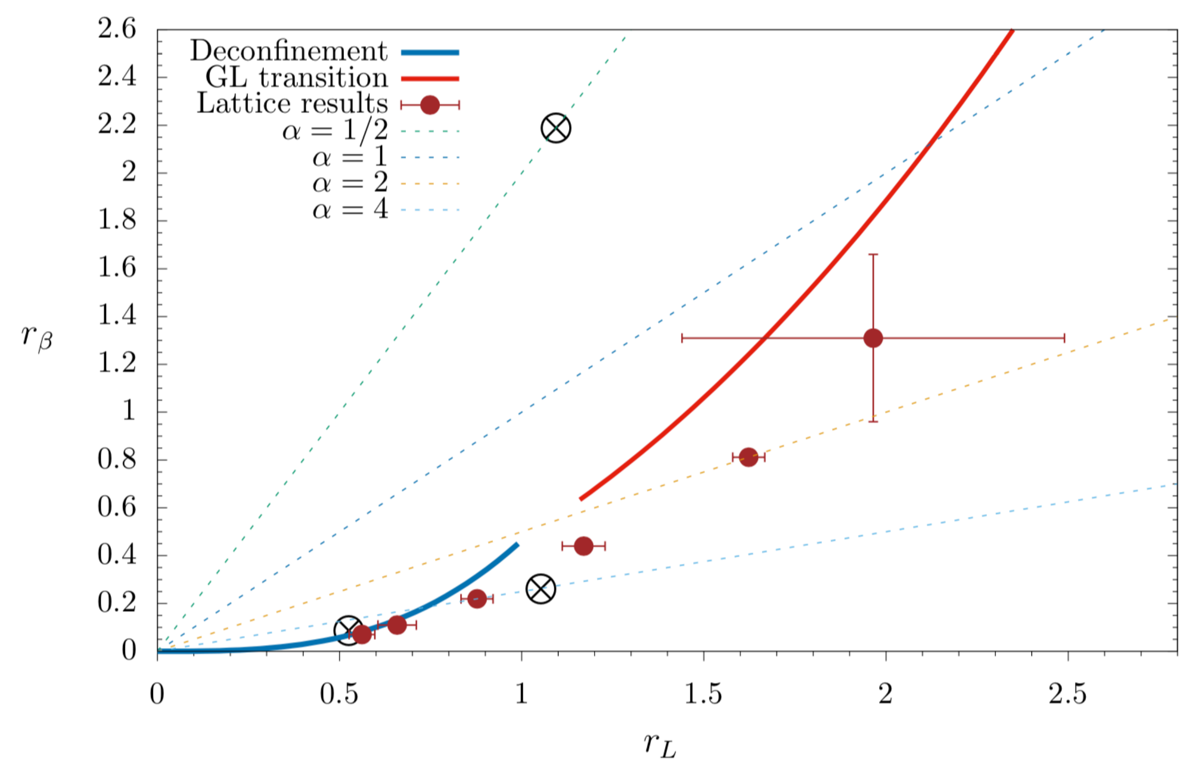
2. The string coupling should be small i.e. $g_s \to 0$

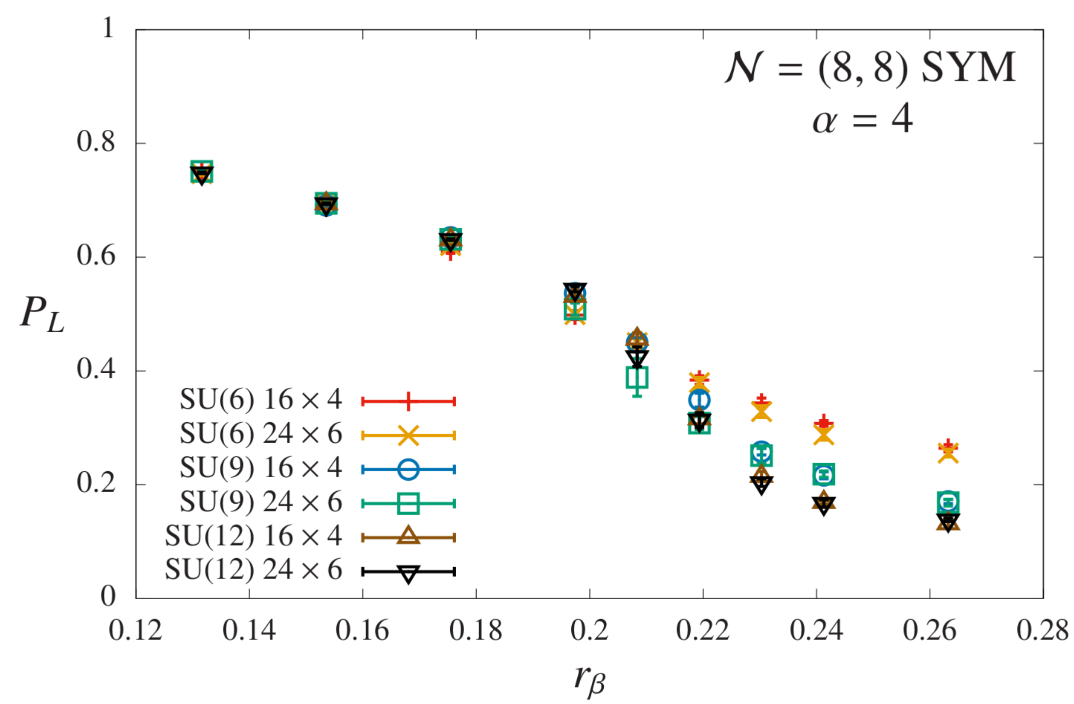
Both these conditions combined gives: $1 \ll r_\beta \ll N^{2/3}$

We could at best reach lattice of about ~250 sites with $SU(16)$ and different α

Results [PRD, 2018]





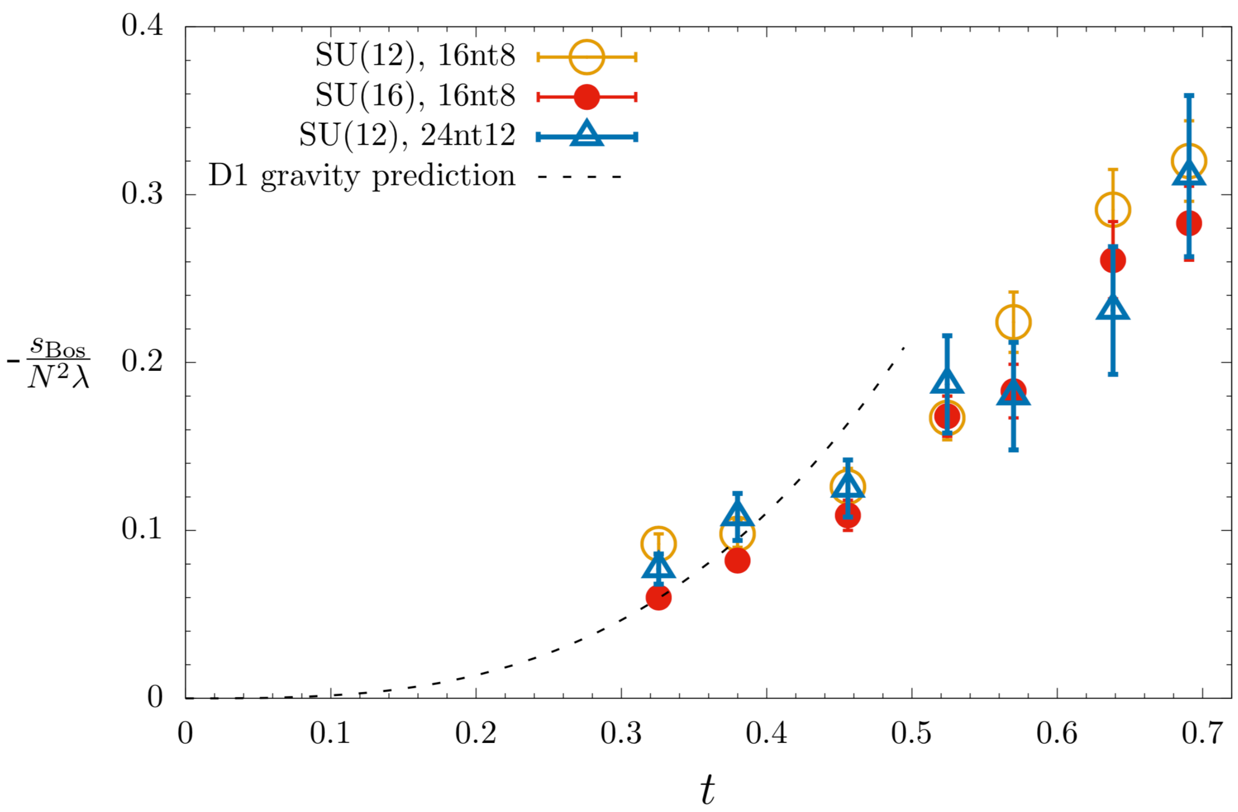


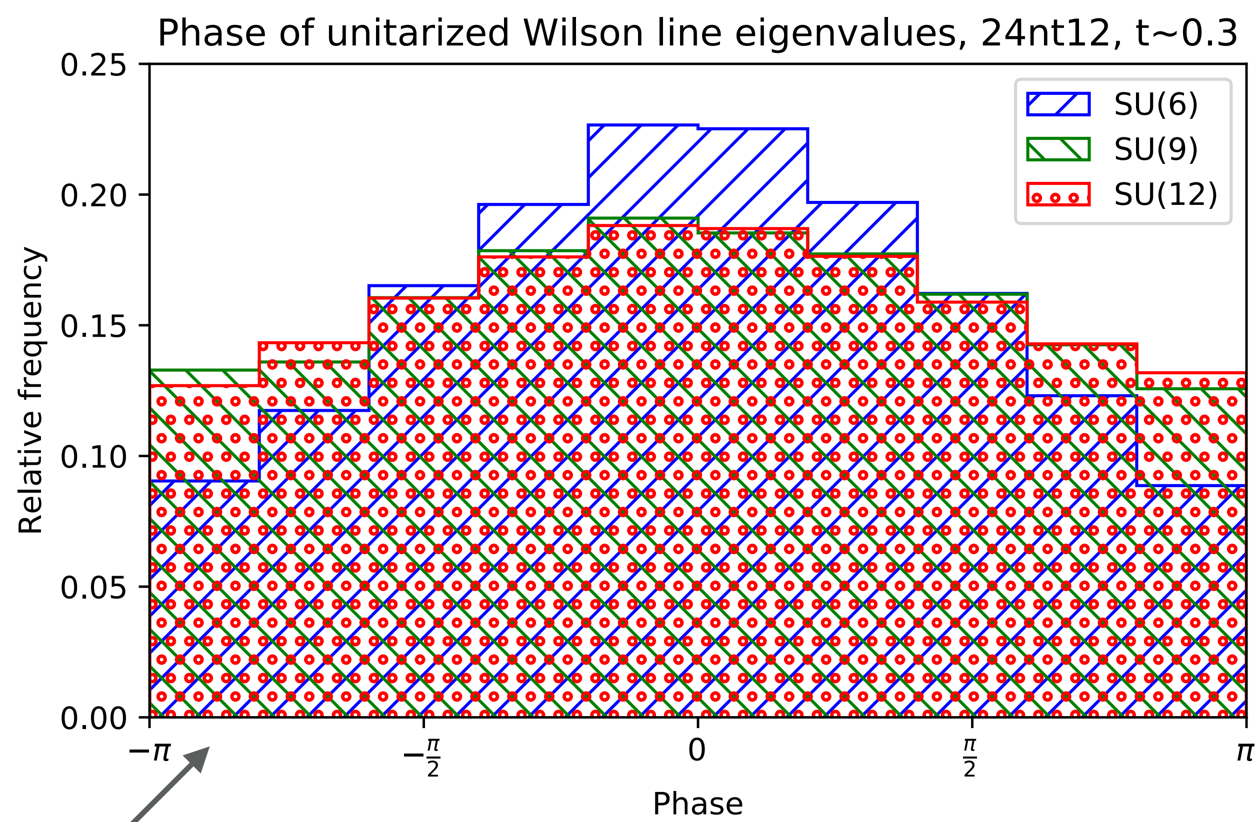
Thermal behaviour in different phases

Uniform black-string phase has action density computed by gravity computations given by $-\frac{s_{\text{Bos}}}{N^2\lambda} = -1.728/r_\beta^3$ while the localized phase has the gravity result $-\frac{s_{\text{Bos}}}{N^2\lambda} = -\frac{2.469}{r_\beta^{16/5}\alpha^{2/5}(1-\gamma^2)^{7/5}}$.

If the lattice was not skewed (reduction from A_4^* to A_2^*) and basis vectors orthogonal then $\gamma = 0$ while for triangular lattice it is $\gamma = 1/2$. It is then a challenge for lattice to reproduce these results.

Results [published in PRD, 2018]





Uniform D1-phase

2+1-dimensional SYM

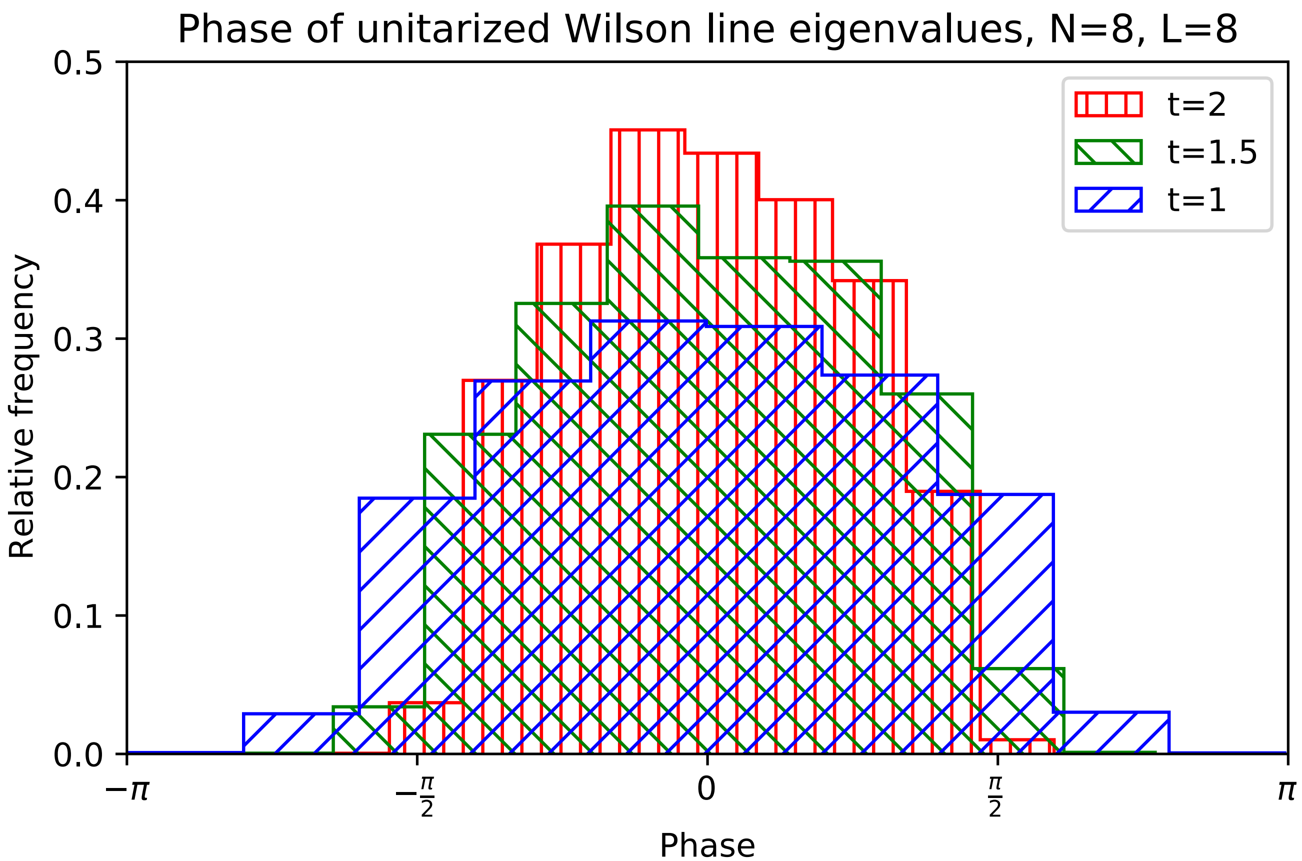
The three-dimensional SYM also has a holographic description at large N and strong coupling. In this case, the black holes are known as black $D2$-branes. The weak coupling (high-temperature) thermodynamic behaviour is just expected to be $\sim t^3$ by just counting d.o.f. while the power-law behaviour of the energy density changes at strong coupling.

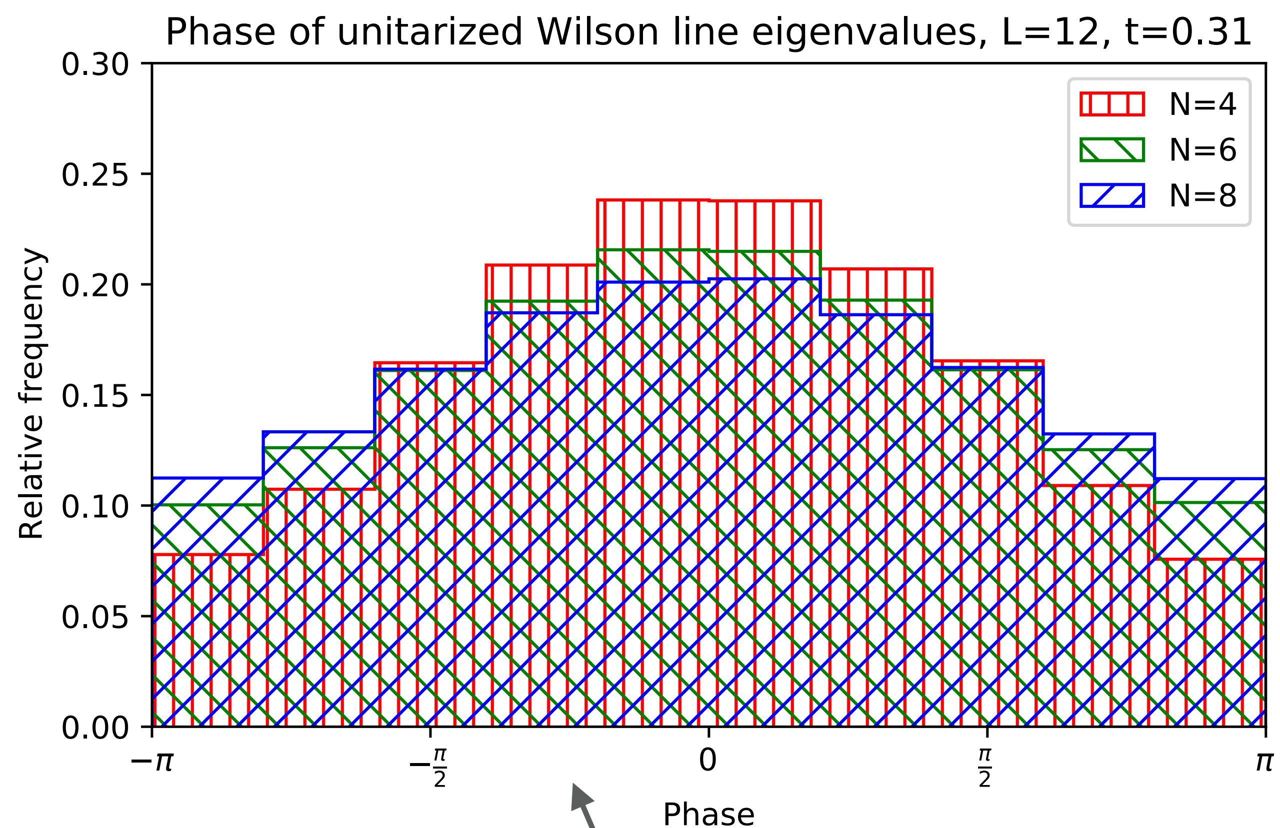
$$\frac{s_{\mathrm{Bos}}}{N^2\lambda^3} = -0.831 t^{10/3} \qquad \frac{s_{\mathrm{Bos}}}{N^2\lambda^3} = -2.598\ldots t^3$$

increasing $t = T/\lambda$

It is worth noting that for SYM on an analogous torus in $p+1$-dimensions we would have parametric dependence $s_{\mathrm{Bos}} \propto t^{(14-2p)/(5-p)}$ for $t \ll 1$ from the gravity dual, and the $t \gg 1$ limit would go as $s_{\mathrm{Bos}} \propto t^{p+1}$. In the $p = 3$ conformal case these powers coincide

Results [published in PRD, 2020]



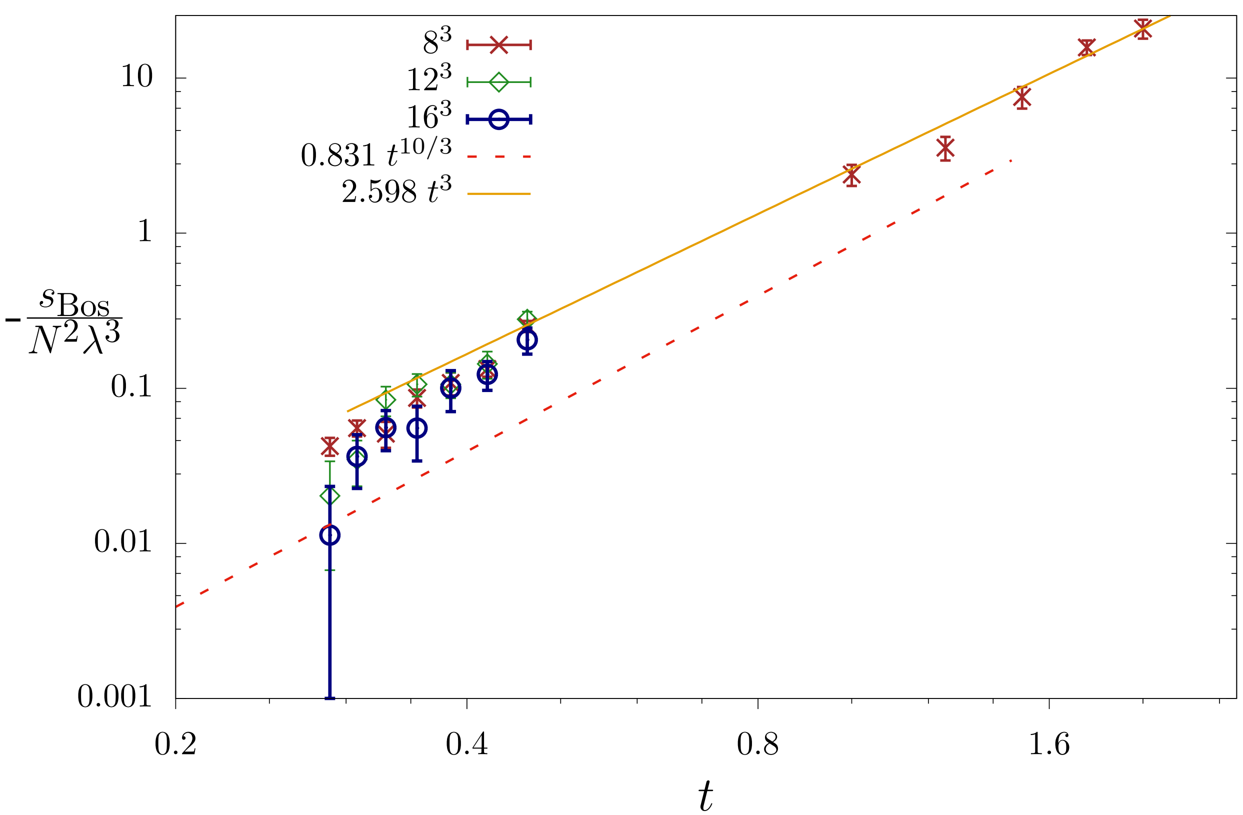


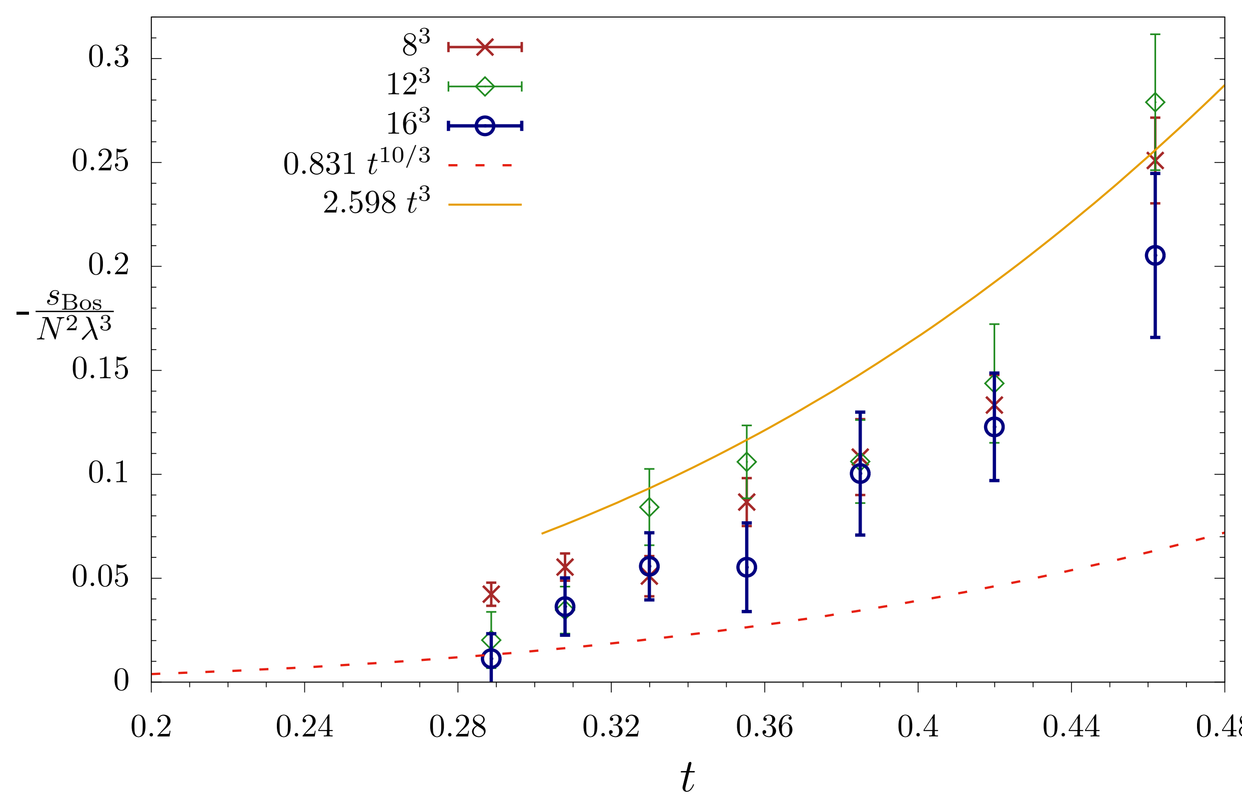
Center symmetry broken phase (high T i.e. λt)

Uniform nature improves with large N. We refer to this as D2 phase.

Results [published in PRD, 2020]

The high-T behaviour serves as a check of our lattice computations. At low temperatures, it starts to tend towards the gravity predictions. These results are for $N = 8$. We have seen that increasing N is as important as taking the continuum limit. These results made use of ~ 5 million core-hours.





Future directions

- The major problem ahead is studying the thermodynamics of $\mathcal{N} = 4$ SYM in four dimensions. The temperature dependence is trivial since it is always $\sim T^4$ but coupling dependence is unknown since there is no tool available at finite λ.
- It would be interesting to study the Maldacena-Wilson loop in these supersymmetric gauge theories which are defined as:

$$W = \frac{1}{N} \mathrm{Tr}\ \hat{P} \exp \left[\oint_{C=\sigma^\mu(s)} ds \Big(A_\mu(\sigma) \dot{\sigma}^\mu + \hat{\theta}^i(s) X_i(\sigma) \Big) \right]$$

- Study static potential in $\mathcal{N} = 4$, scaling dimensions of some simple BPS not-protected operators.
- Computational: Parallelize over matrix degrees of freedom!

Thank you