NUMERICAL APPROACHES TO HOLOGRAPHY

Talk online at - raghavgjha.net

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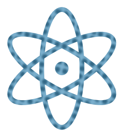
(from 01.09.2019)

Overview

1. Holographic principle (Gauge/gravity duality)
2. Connection between tensor networks and holography
3. Tensor formulation of 2d gauge-Higgs system.
4. Lattice studies of supersymmetric field theories
5. Conclusion

Remarks!

The numbers quoted for reference are the arXiv numbers. You can search for those on www.arxiv.org 

SYM = Supersymmetric Yang-Mills. An example of quantum field theory which is supersymmetric (i.e. the bosonic terms and fermionic terms are related by some symmetry). This is a very special property. No experimental evidence yet.

Quantum Field theory — Combination of classical field theory, special relativity, and quantum mechanics. For ex. - QED (Quantum Electrodynamics) is a relativistic quantum field theory of electrodynamics and deals with interactions of photons with matter.

Gravity!

We have a fantastic theory of classical gravity known as 'General Relativity (GR)'. Several checks over the past 100 years, most recent being the one made by LIGO (Laser Interferometer Gravitational-Wave Observatory).

But, not a quantum theory! Quantum gravity become important to understand effects of black holes/Big bang. Dimensional analysis shows,

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \sim 1.6 \times 10^{-35}\mathrm{m}$$

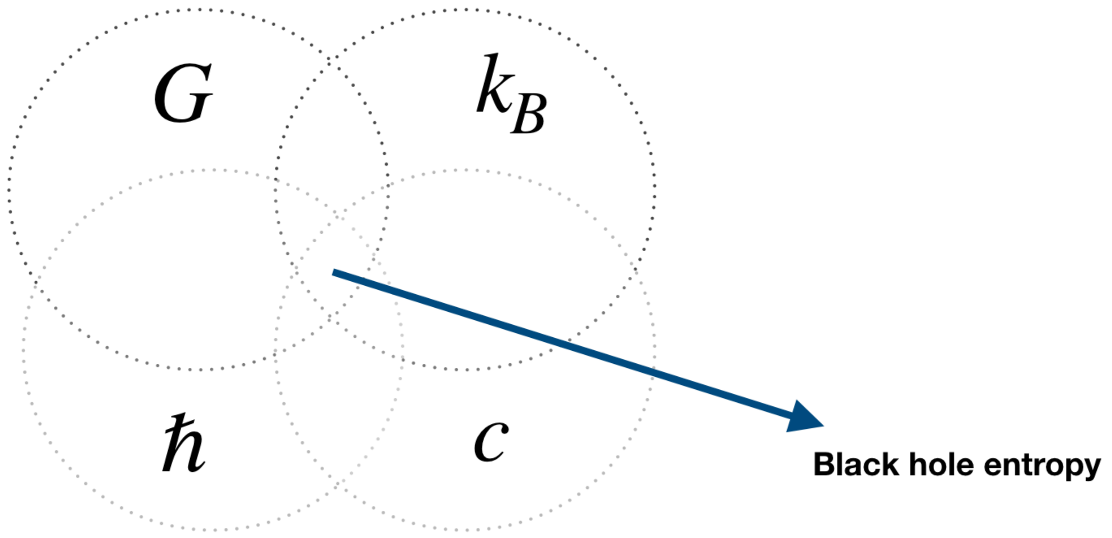
What is "holography" ?

The idea that a gravitational theory in one higher dimension (d+1) is related to some quantum field theory (without gravity) in one lower dimension (d) on its boundary.

It is expected that the theory of quantum gravity will admit a holographic description.

First hints came in 1970s, when Stephen Hawking and Jacob Bekenstein found that the black hole entropy was proportional to the area of its event horizon.

$$S = \frac{k_B c^3 A}{4G\hbar}$$



dS/AdS

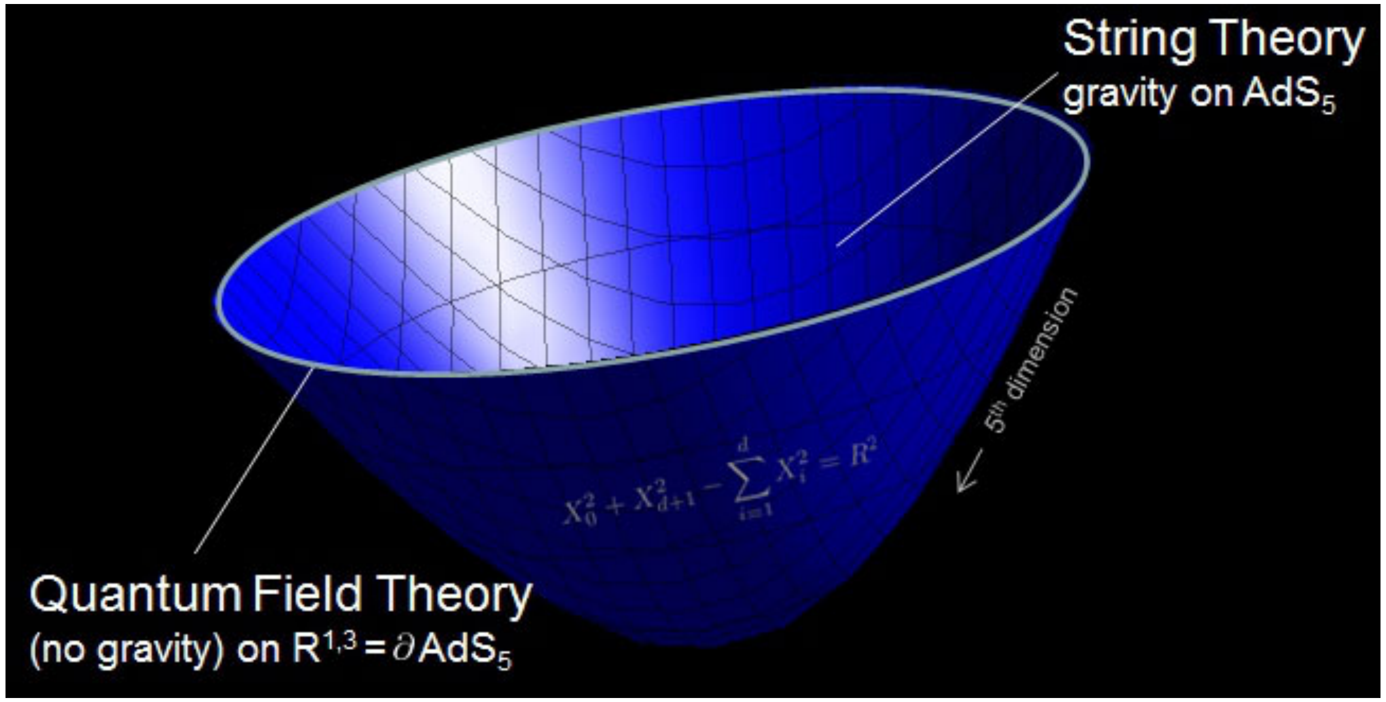
Positive vacuum energy implies positive curvature (de Sitter space). de Sitter spacetime is the maximally symmetric spacetime of constant positive curvature. D-dimensional de Sitter spacetime can be viewed as being embedded in D+1-dimensional Minkowski spacetime.

Like a spherical surface => positive curvature => no boundaries **Difficult**

Negative vacuum energy is negative curvature (anti-de Sitter space = AdS). It is the maximally symmetric spacetime of constant negative curvature.

AdS/CFT (Gauge/gravity)

A concrete example of two systems holographically related to each other.



Lower dimensions

- Maximally supersymmetric Yang-Mills theory in p+1-dimensions is dual to Dp-branes* in supergravity at low temperatures in a special limit (large N, strong coupling). [N is the size of matrix]

- In fact, holographic behaviour have been seen even in systems with no SUSY, for ex. CFT_2/AdS_3.

* A D0-brane is a single point, a D1-brane is a line, a D2-brane is a plane. A p-brane sweeps out a (p+1)-dimensional volume in spacetime called its world-volume.

Weak/strong conjecture

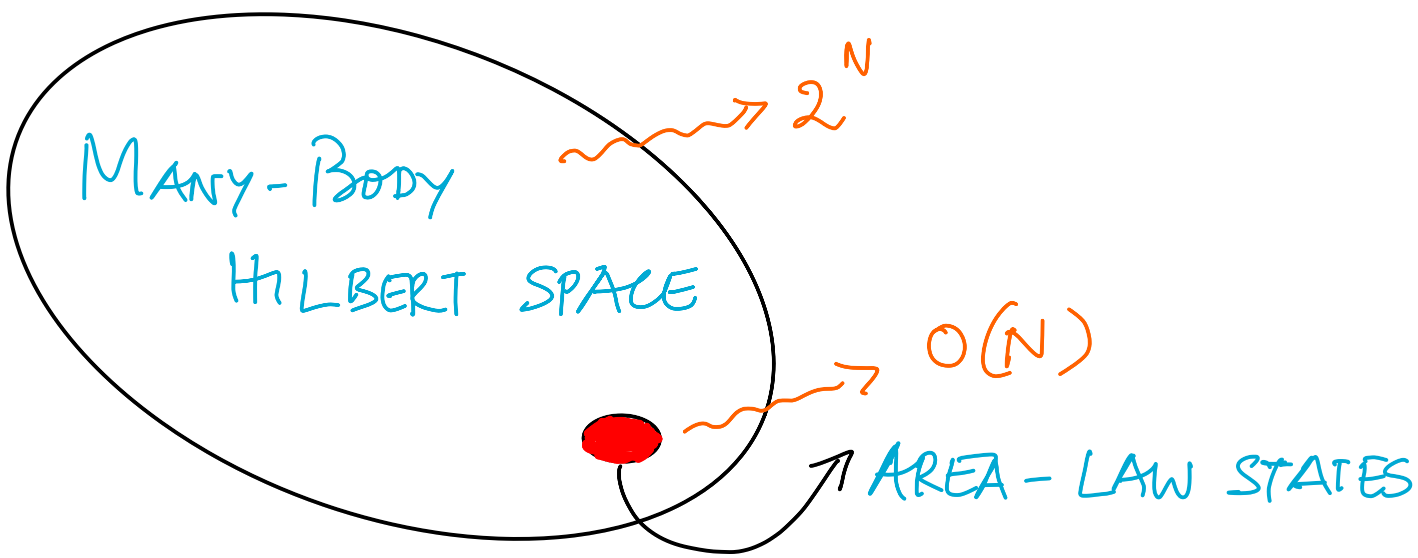
Even though AdS/CFT or more generally gauge/gravity is a wonderful idea connecting two different theories in different dimensions, it has no proof. Need to rigorously test this conjecture has led to lot of interesting work.

Tensor networks

Several motivations to study tensor networks

1. Expected to play fundamental role in putting AdS/CFT on a firm footing via understanding the properties (geometry) of bulk physics from entangled quantum state. ***Brief mention***
2. Provides an arena for studying lower-dimensional critical systems (faster and more efficiently) than any other methods.
3. Possibility of formulating non-holographic gauge theories in terms of tensors can enable us to study theories where usual Monte Carlo methods fail [sign problems!] -> ***Some results***

Hilbert space - too big!



if N= 10^{23} (Avogadro Number), then number of basis states is more than atoms in universe! Luckily for us, not all quantum states are important! How to find nice ones is a big and interesting problem.

Many-body ground state

Ground states are not "arbitrary" states in Hilbert space, it has very special features. Some of which have been captured by studying the entanglement entropy (EE). The region of Hilbert space that obeys area-law scaling for the EE corresponds to a tiny corner (in red). Therefore, lot of progress have been made in many-body physics by finding EE and hence identifying important regions of Hilbert space.

Classes of local Hamiltonian

$\mathscr{H}$	$C(x_1, x_2)$	$S(A)_{1+1\text{d}}$
Gapped / Non-Critical	$\sim \exp[(x_1 - x_2)/\xi]$	$\sim L^0 \sim$ constant
Gapless / Critical	$\sim \vert x_1 - x_2 \vert^{-q}$	$\ln(L_A)$

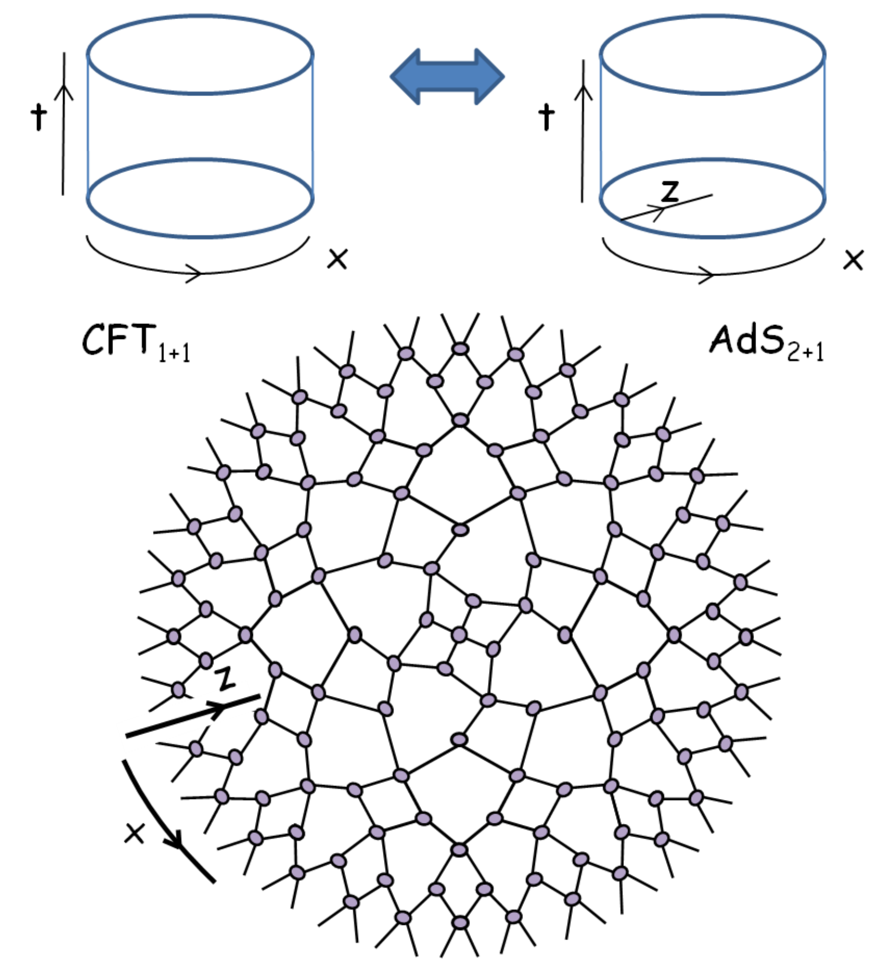
S is the entanglement entropy at T=0 , $\xi \geq 0$ is the correlation length

AdS/MERA

[Swingle, 2009]

Based on these observations, several tensor networks have been proposed which tries to capture the ground state wave function of the system near critical point or away from it. One such tensor network which is used to describe critical systems (with 'log' scaling of EE) is called multi-scale entanglement ansatz (MERA) and it efficiently captures the ground state of the critical systems (which are CFTs in field theory language).

(Figure: **1106.1082**)



Representation of ground state wave function

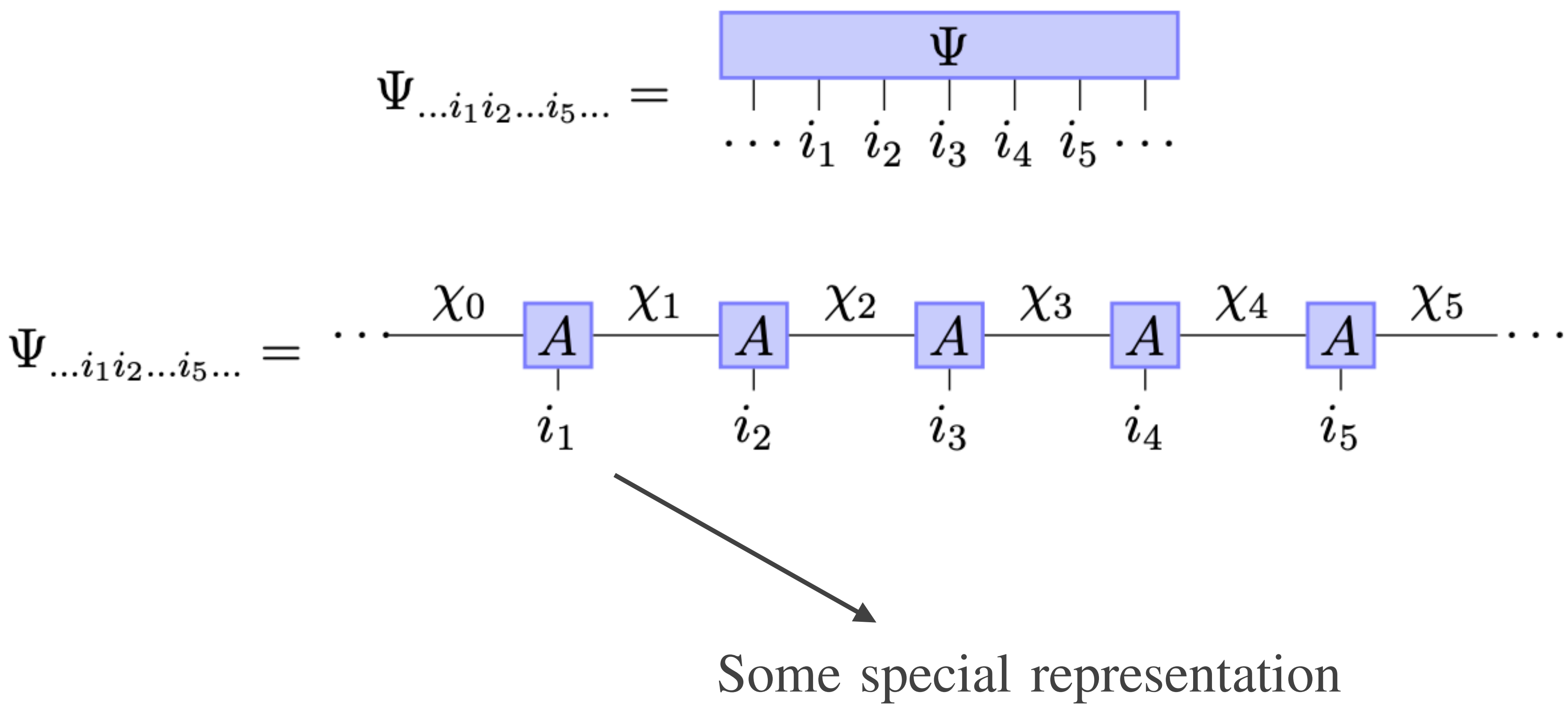


Fig. from **1811.11027**

Basics of Tensor indices!

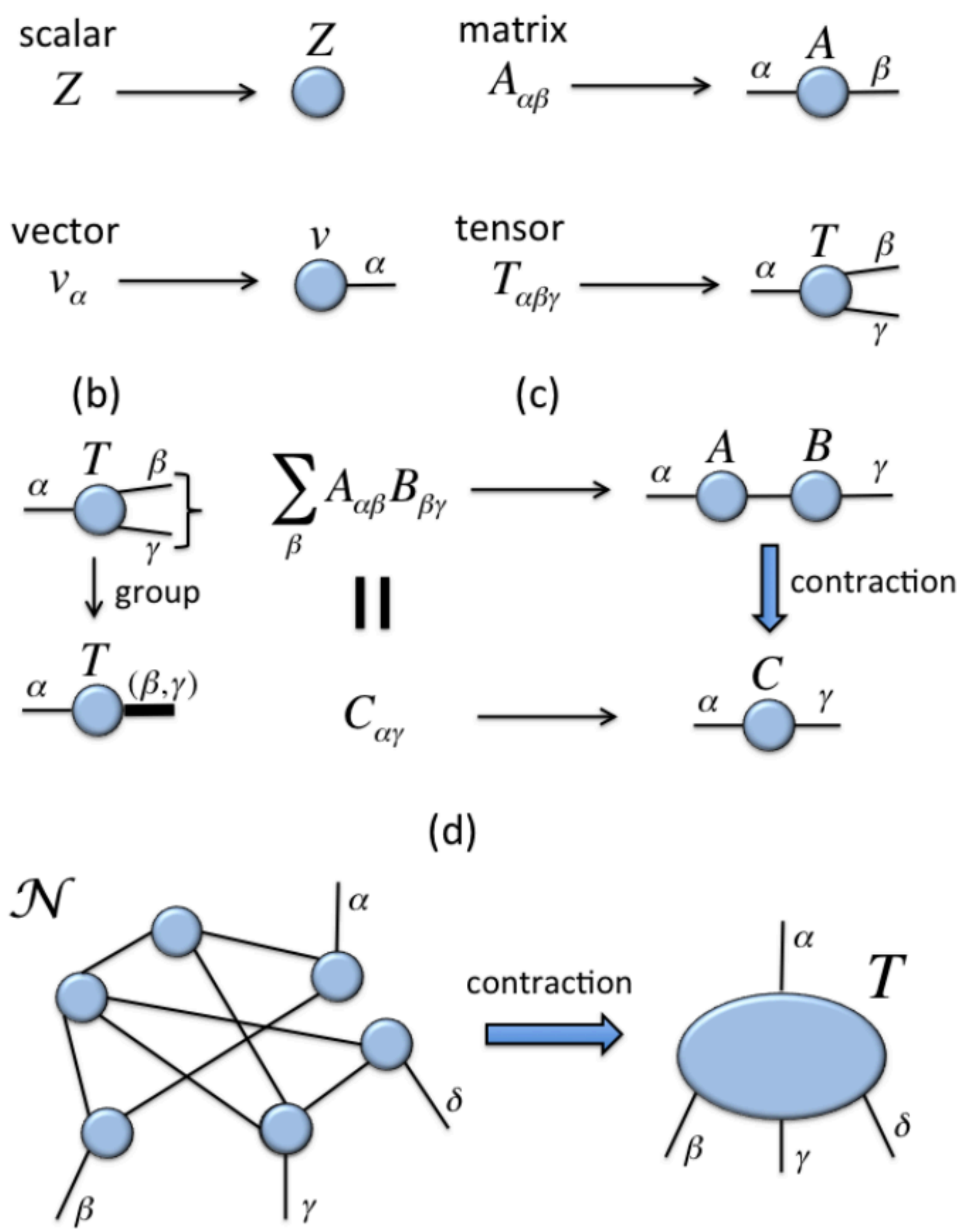
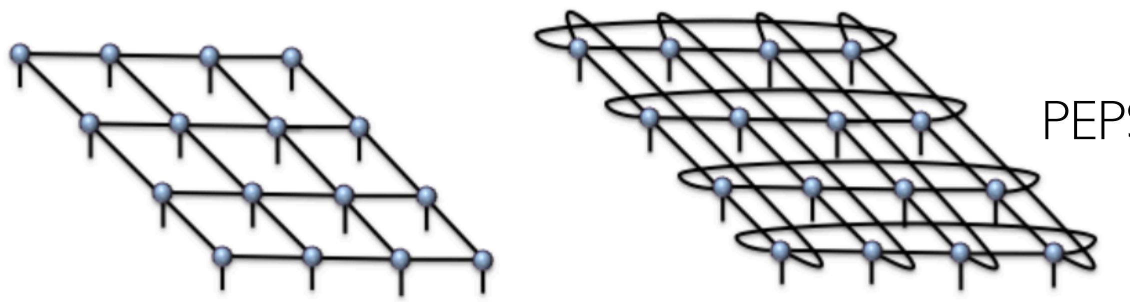


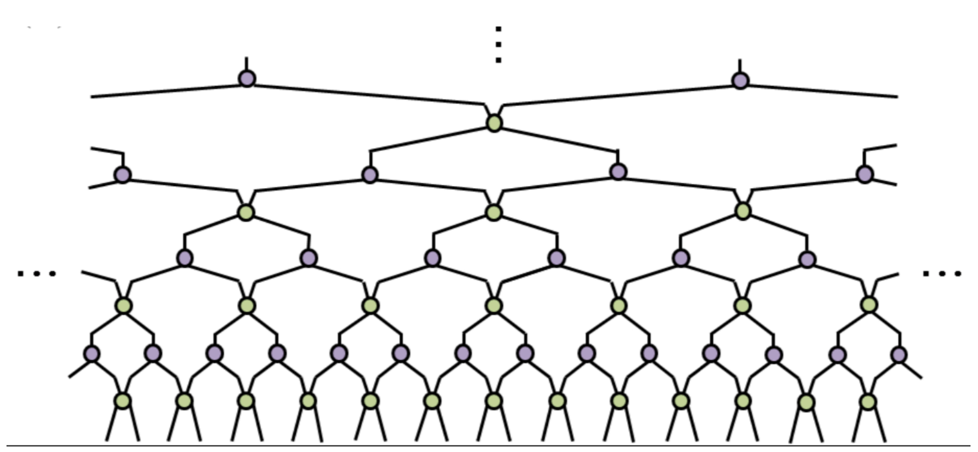
Figure: 1112.4101

Different tensor networks

MPS (Matrix Product States) with open boundary condition (OBC) and periodic boundary conditions (PBC), Figure: **1306.2164**



PEPS (Projected Entangled Pair States)



Scale invariant MERA, Figure: **1106.1082**

Green := Disentangler

Violet := Coarse graining tensor

2d non-Abelian gauge+Higgs model

(A Bazavov, S Catterall, RGJ, J Unmuth-Yockey, Phys. Rev. D 99, 114507 (2019), **1901.11443**)

$S = \left(\frac{-\beta}{2} \mathrm{Tr}\Box - \frac{\kappa}{2} \mathrm{Tr} U \right)$, where β is the gauge coupling and κ is the matter coupling in the unitary gauge. The first term is the standard pure gauge Wilson action featuring a plaquette.

We expand the Boltzmann weights in terms of characters (called character expansion). Writing, $S = S_g + S_\kappa$

$$e^{-S_g} = \prod_x \sum_r F_r(\beta) \chi^r(UUU^\dagger U^\dagger)$$

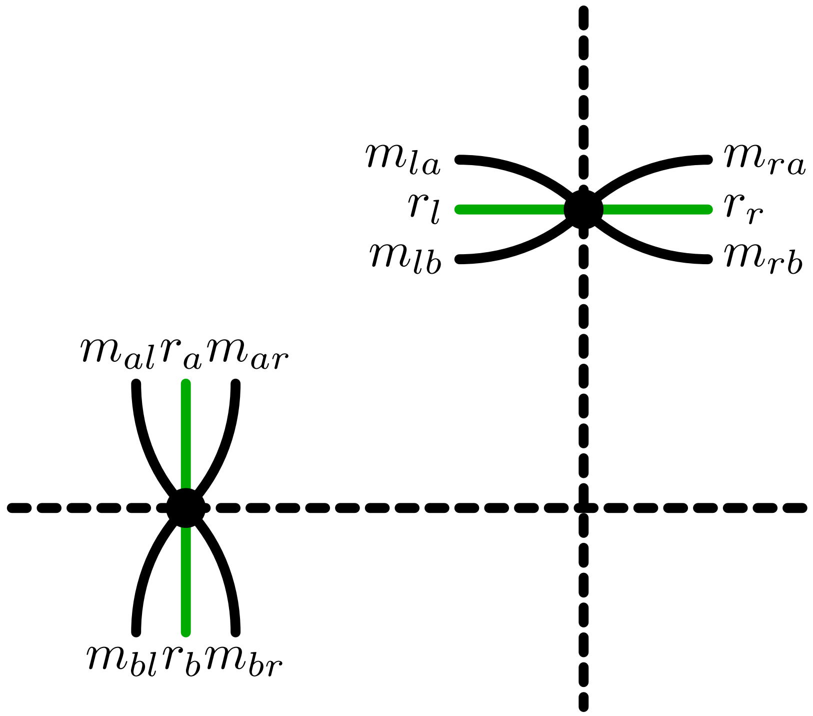
Link (A) and plaquette (B) tensors

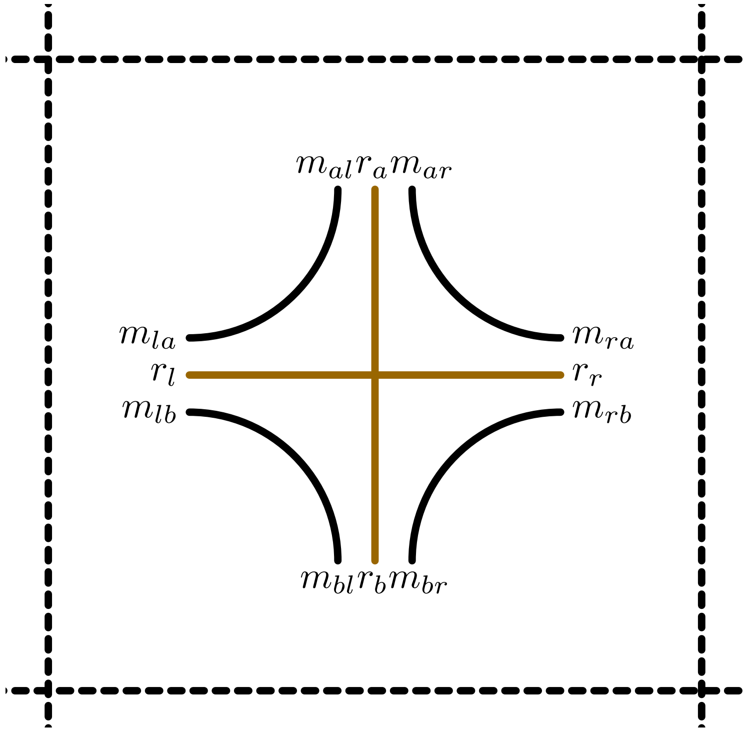
$$A_{(r_l m_{la} m_{lb})(r_r m_{ra} m_{rb})}(\kappa) = \frac{1}{d_{r_r}} \sum_{\sigma=|r_r-r_l|}^{r_r+r_l} F_\sigma(\kappa) C^{r_r m_{rb}}_{r_l m_{lb} \sigma (m_{rb}-m_{lb})} \times C^{r_r m_{ra}}_{r_l m_{la} \sigma (m_{rb}-m_{lb})} .$$

$$B_{(r_l m_{la} m_{lb})(r_r m_{ra} m_{rb})(r_a m_{al} m_{ar})(r_b m_{bl} m_{br})} = \begin{cases} F_r(\beta) \quad \delta_{m_{la},m_{al}} \delta_{m_{ar},m_{ra}} \delta_{m_{rb},m_{br}} \delta_{m_{bl},m_{lb}} & \text{if } r_l = r_r = r_a = r_b = r \\ 0 \qquad \text{else.} \end{cases}$$

With the knowledge of these two tensors, one can construct the fundamental tensor.

Diagrammatic representation of A and B tensors

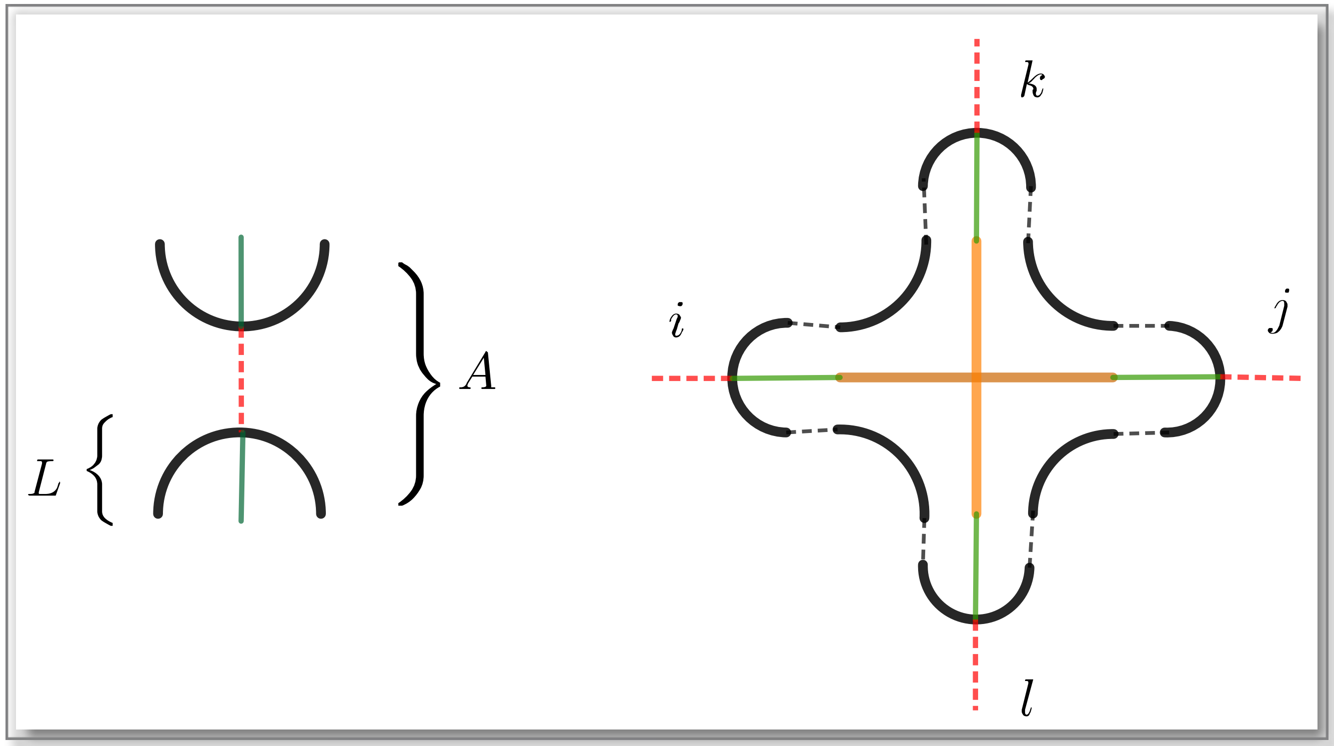




The fundamental tensor (T)

Using A and B tensor, we can construct the fundamental tensor. And then with several copies of this tensor we can make up the entire lattice. Note that we can just use one A and B by exploiting the translational invariance of the lattice.

$$T_{ijkl}(\beta,\kappa) = \sum_{\alpha,\beta,\gamma,\delta} B_{\alpha\beta\gamma\delta}(\beta) L_{\alpha i} L_{\beta j} L_{\gamma k} L_{\delta l}(\kappa), \quad \text{where} \quad A = LL^T$$



Coarse-graining the tensor network

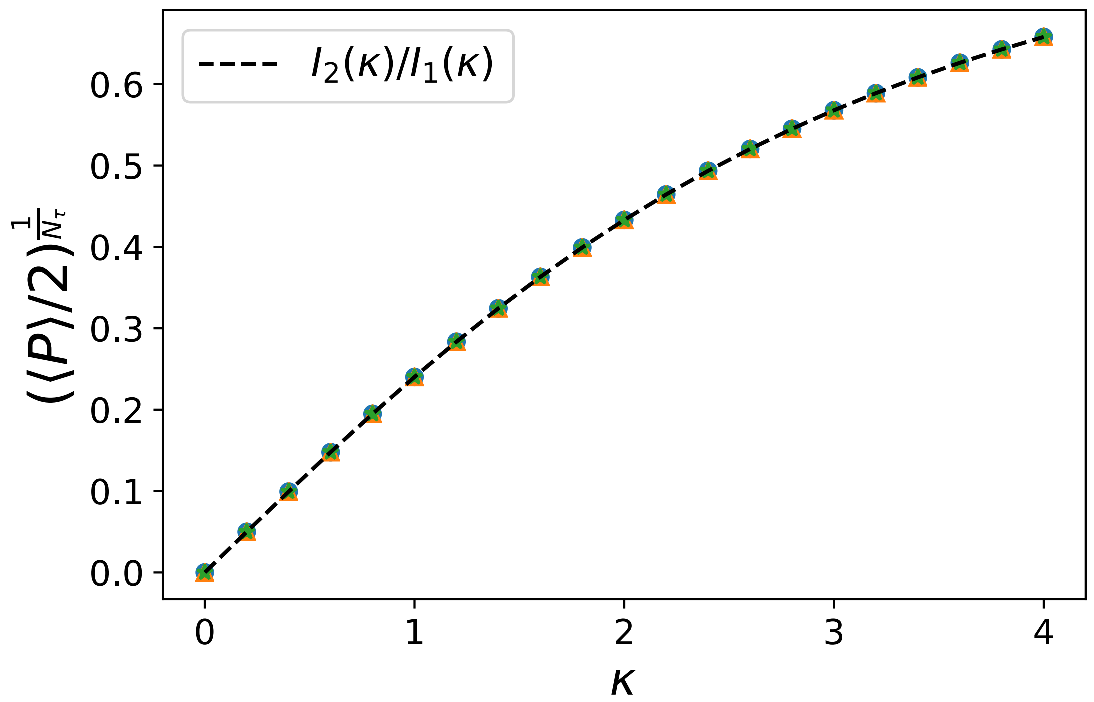
We use to HOTRG (higher order tensor renormalisation group) to implement coarse-graining and truncate the local space to $D=50$, and we use r_max = 1, which corresponds to $T = 14^4$. Using r_max = $\frac{1}{2}$, will give us $T = 5^4$. The choice of r_max depends on the gauge theory one wants to study and the coupling. For ex- sometimes when the coupling is strong, the sum over "irreps" converges very fast.

We can write the partition function as translation invariant tensor network state as,

$$Z = \mathrm{Tr} \prod T_{ijkl}$$

Exact results for $\beta = 0$ and $\kappa = 0$

For $\beta = 0$, we can write exact value of the Polyakov loop in terms of Bessel functions. This provides a simple check of the tensor formulation.

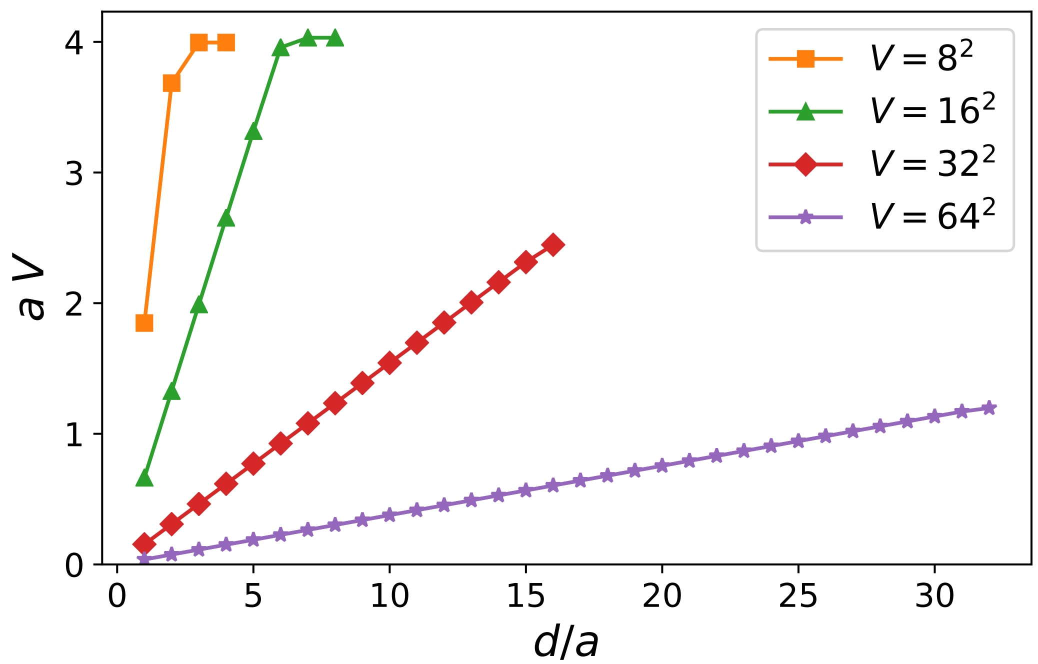


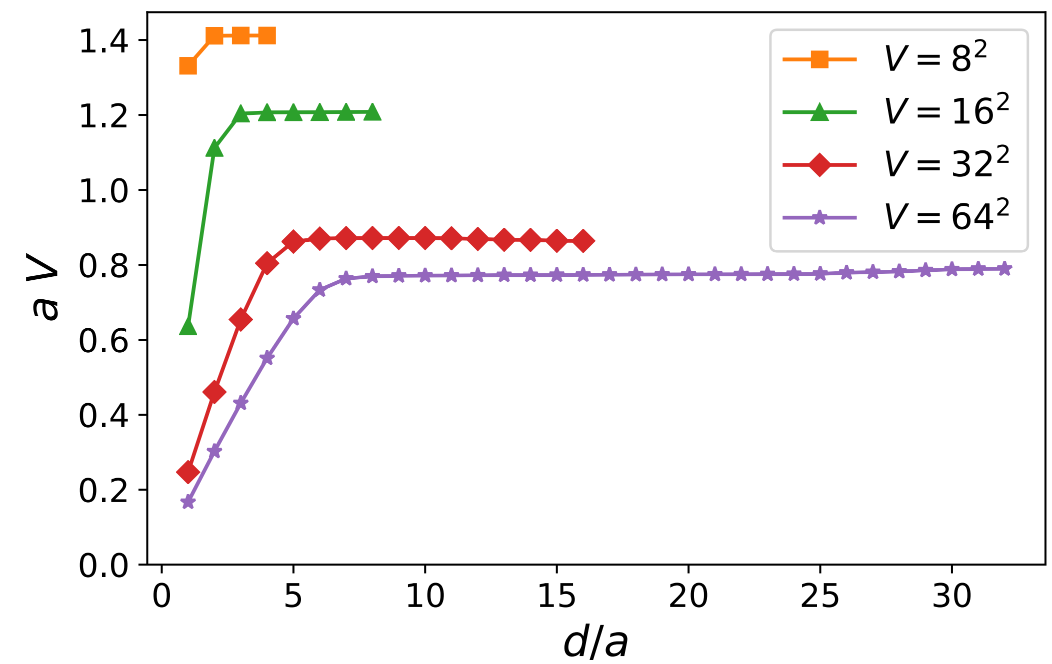
Confining & Higgs Phase

In the limit of zero temperature, the Polyakov loop correlator is given by, $C(d) = \exp(-\beta V(d))$. This also provides a measure of monitoring confinement when $V \propto d$ (the slope gives σ, string tension). In a Higgs phase, it is constant and independent of R.

Two phases, 1) Confining phase, 2) Higgs-phase.

Results - Static potential

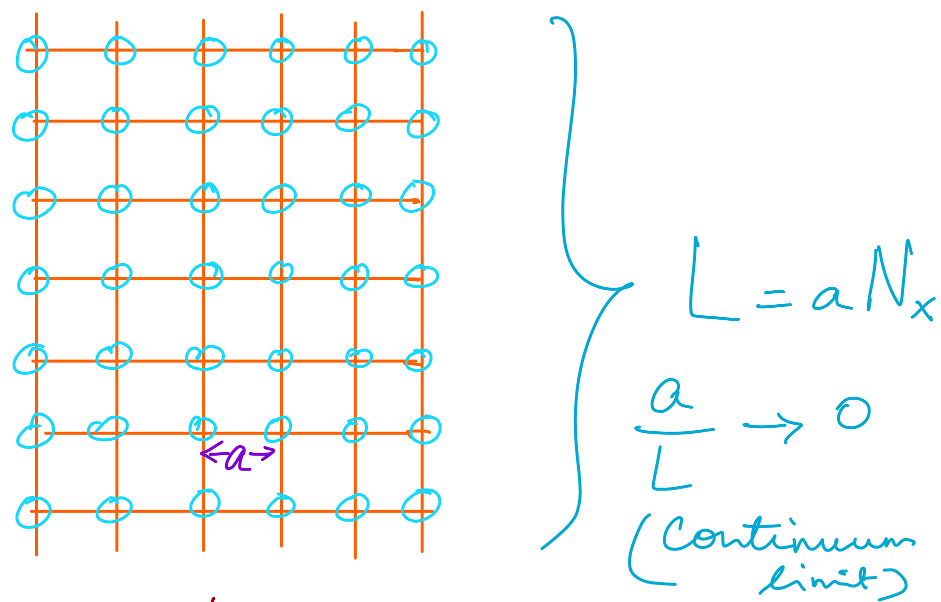




On the left, we have $\kappa = 0.50$ which is in the confining regime and on the right, we have $\kappa = 2.0$ which is in the Higgs phase. There is a cross over around $\kappa \sim 1.3$

One slide review - Lattice calculations

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}\varphi \; O(\varphi) \, e^{-S[\varphi]}$$



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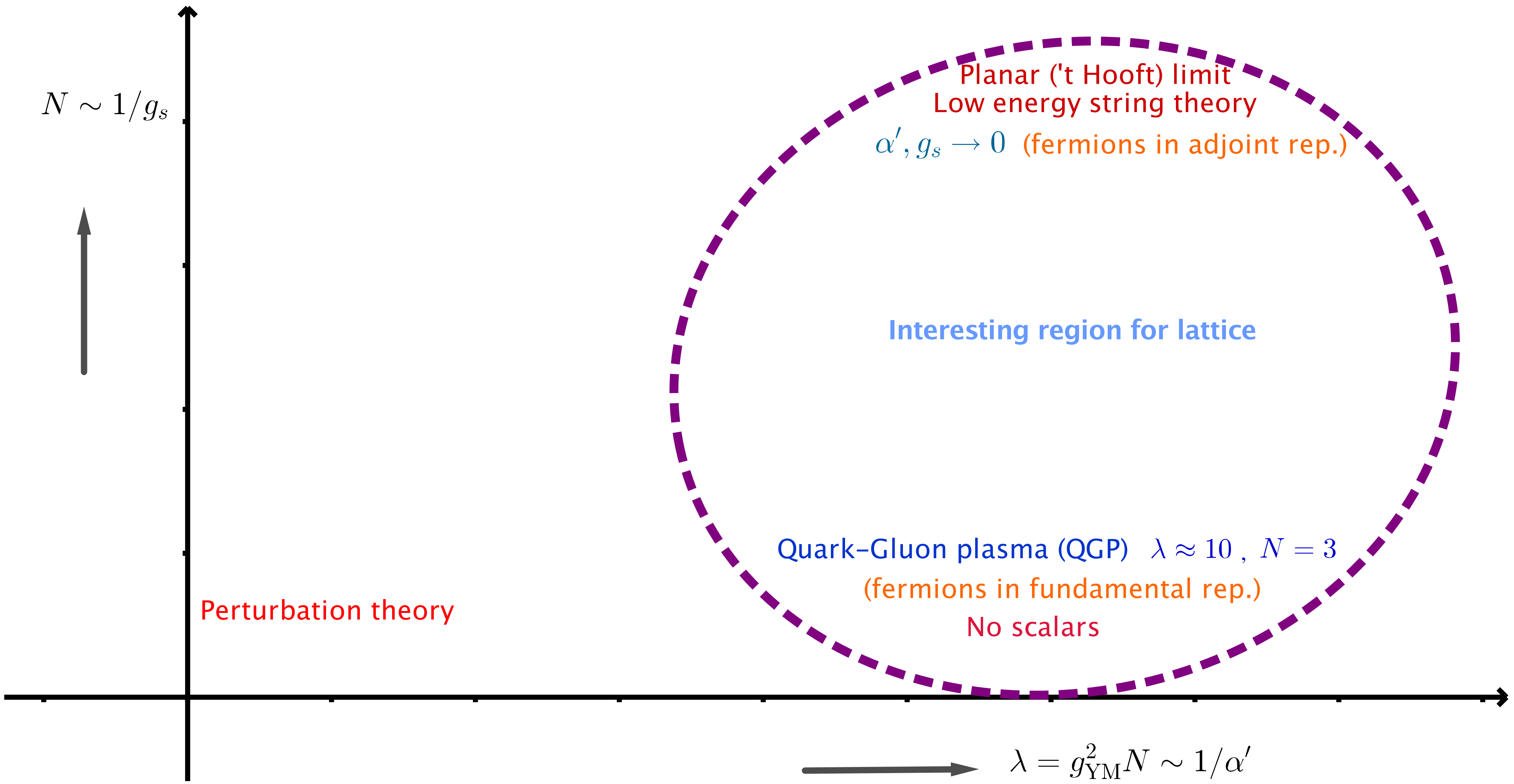
USE IMPORTANT SAMPLING MONTE-CARLO.

Phase diagram for SYM/QCD



$\mathcal{N}$=4 SYM theory

Obtained by dimensionally reducing the ten-dimensional SYM theory down to four dimensions. Conformal field theory, beta-function vanishes, consists of six scalars, sixteen real fermions, all massless and in the adjoint representation of the SU(N) gauge group as N x N matrices.

Most of the holographic predictions are related to this theory or its dimensional reductions. It is important to study this theory numerically in the strong coupling limit. This has led to the field of lattice $\mathcal{N} = 4$ SYM. For a review see, **0903.4881**

Further dimensionally reduced SYMs

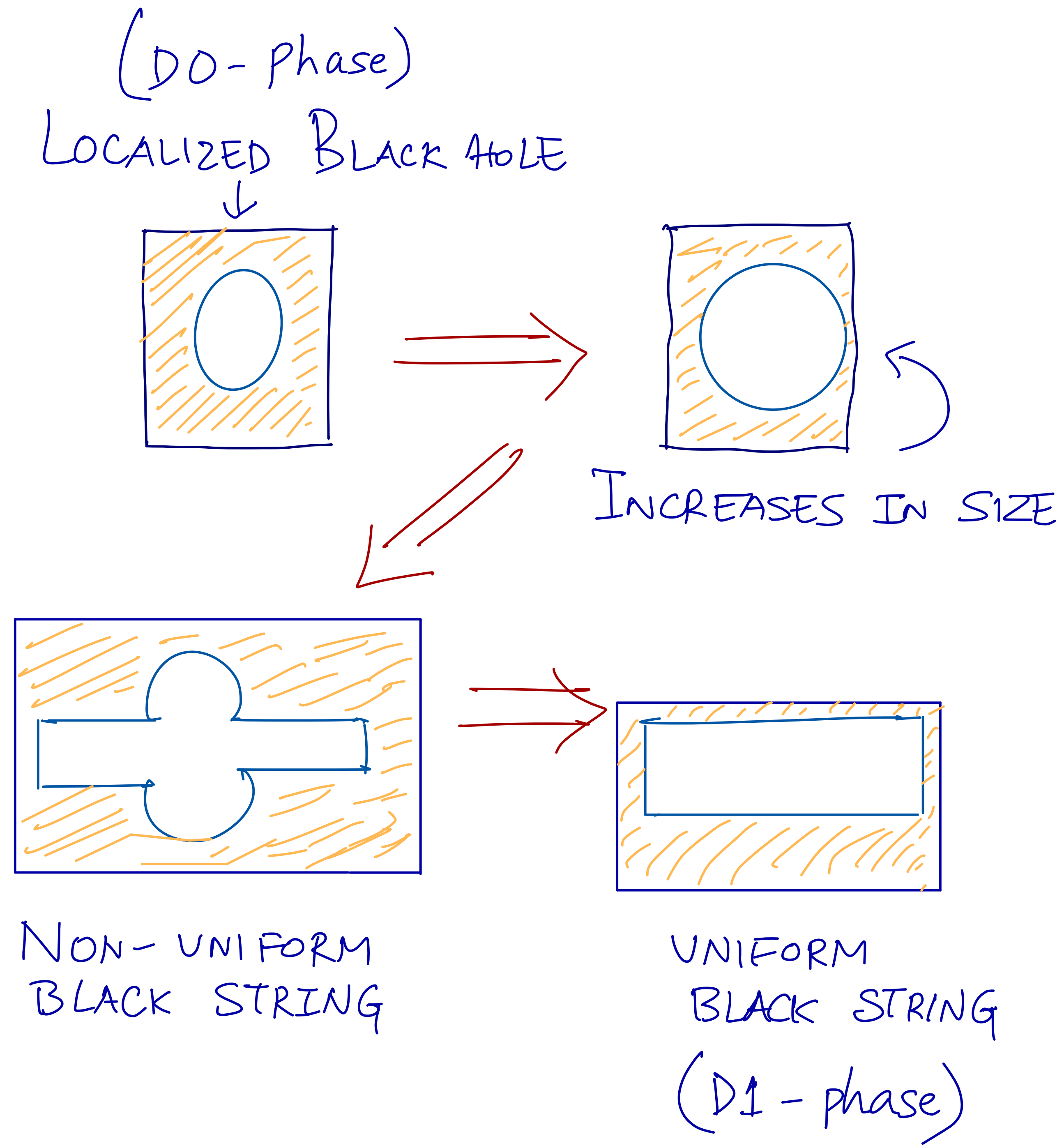
- Reduce the theory to (1+1)-dimensions to study SYM theory dual to SUGRA having different black hole solutions (uniform black string and localised black hole) and transition between them.
- Reduce the four dimensional theory down to one dimension (0+1, time, SQM) to get BFSS (Banks—Fischler—Shenker—Susskind) model.
- Add mass deformation to this theory to study matrix model on pp-wave space-time, known as BMN (Berenstein—Maldacena—Nastase) model.

Two-dimensional maximal SYM

- Dimensionally reduce the four dimensional theory down to two.
- Dimensionless coupling, $r_\tau = \sqrt{\lambda}\beta = 1/t,\ r_x = \sqrt{\lambda}L,\ \ \alpha = L/\beta = r_x/r_\tau$
- Phase transition between localized black hole and black string at $\alpha^2 r_\tau \sim 2.45$
- This is a topological transition on the gravity side, dual to deconfinement transition on the gauge theory side. In the large N limit, but weak coupling, there is well-known 3rd order Gross-Witten-Wadia transition.

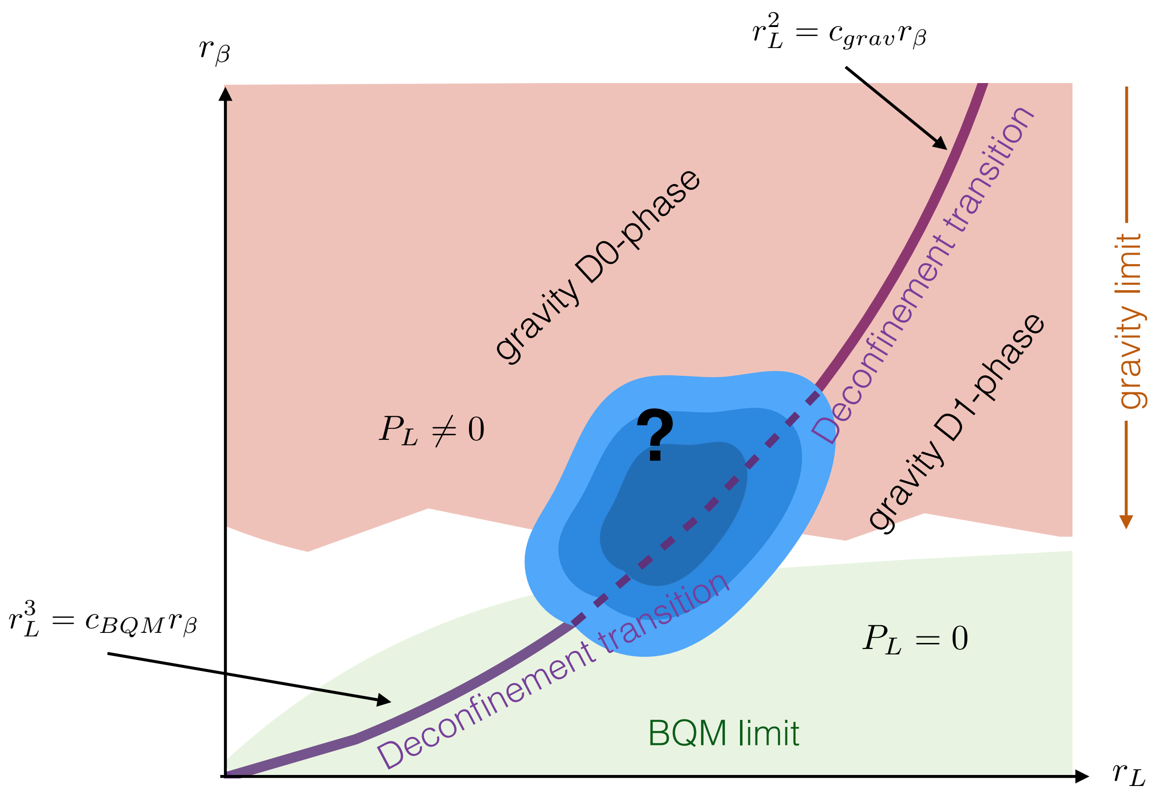
Transition between black hole solutions

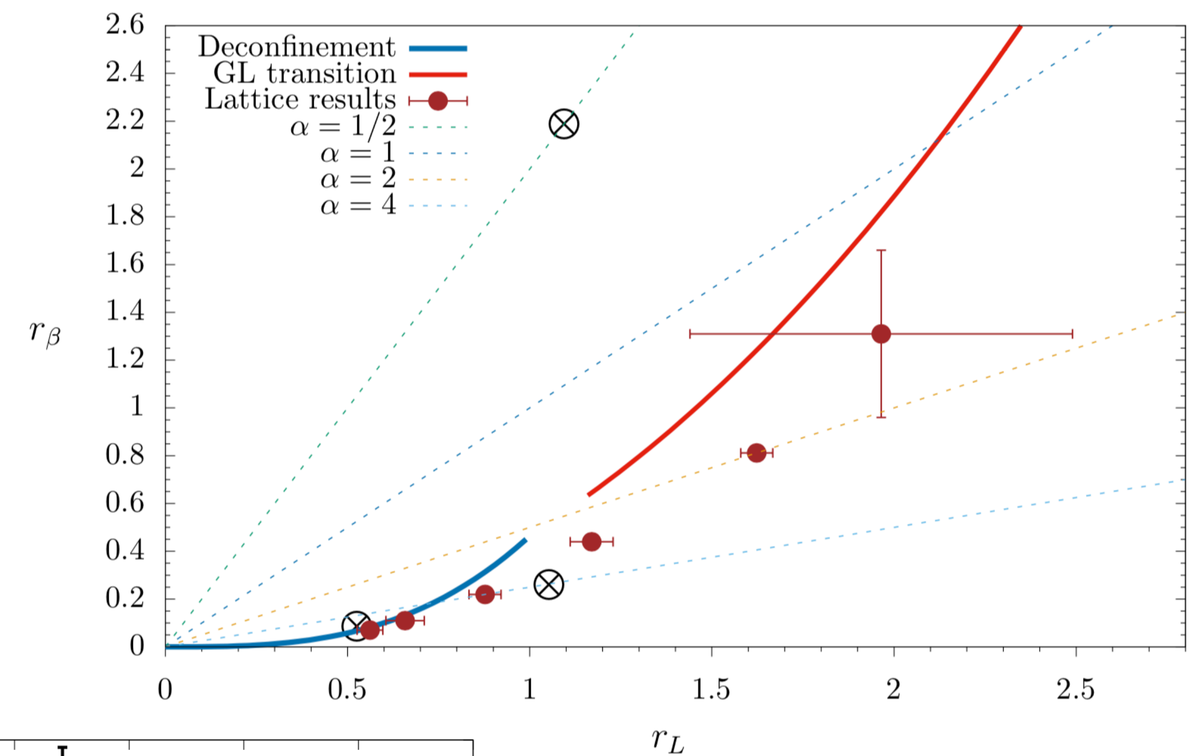
(Micro-canonical ensemble (=fixed energy), solution that maximise S is preferred)

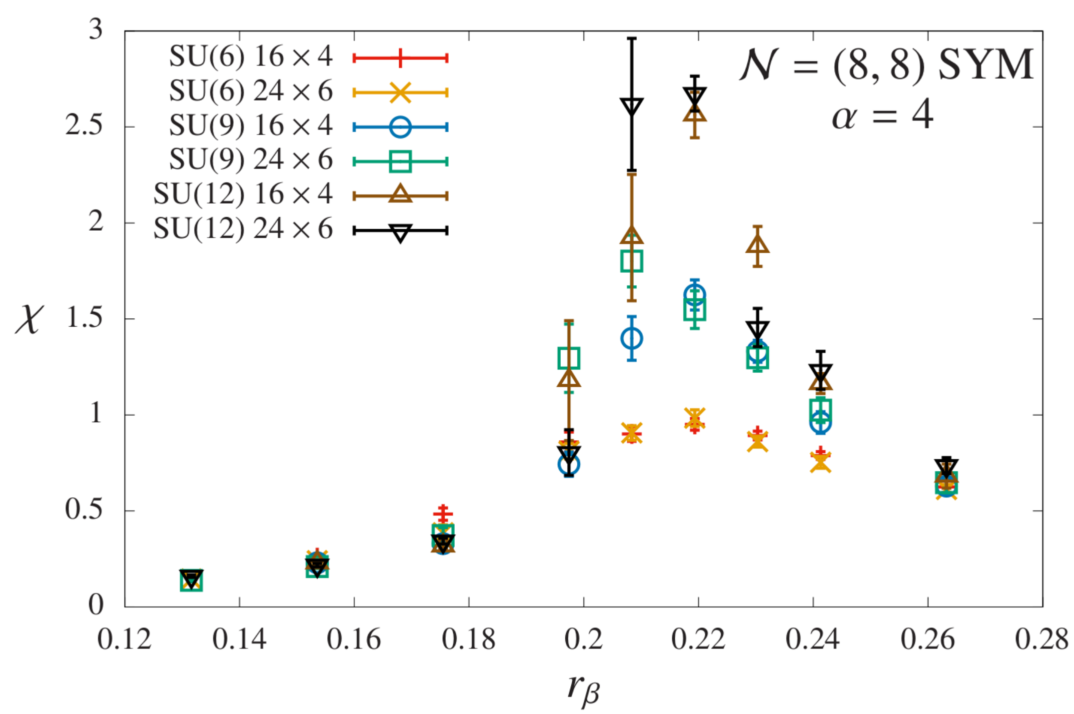


Results

Phys. Rev. D 97, 086020 (2018), **1709.07025**







BFSS matrix model

This model can be obtained by the dimensional reduction of $\mathcal{N} = 1$ SUSY in ten dimensions.

$$S_{\text{BFSS}} = \frac{N}{4\lambda} \int dt \text{Tr}\left[(D_t X^i)^2 - \frac{1}{2}\left[X^I, X^J\right]^2 + \Psi^T D_t \Psi + i\Psi^T \gamma^j [\Psi, X^j]\right]$$

I, J runs from 1 … 9, and we have total of sixteen ψ , where all fields are N x N matrices and in the adjoint representation of the gauge group SU(N).

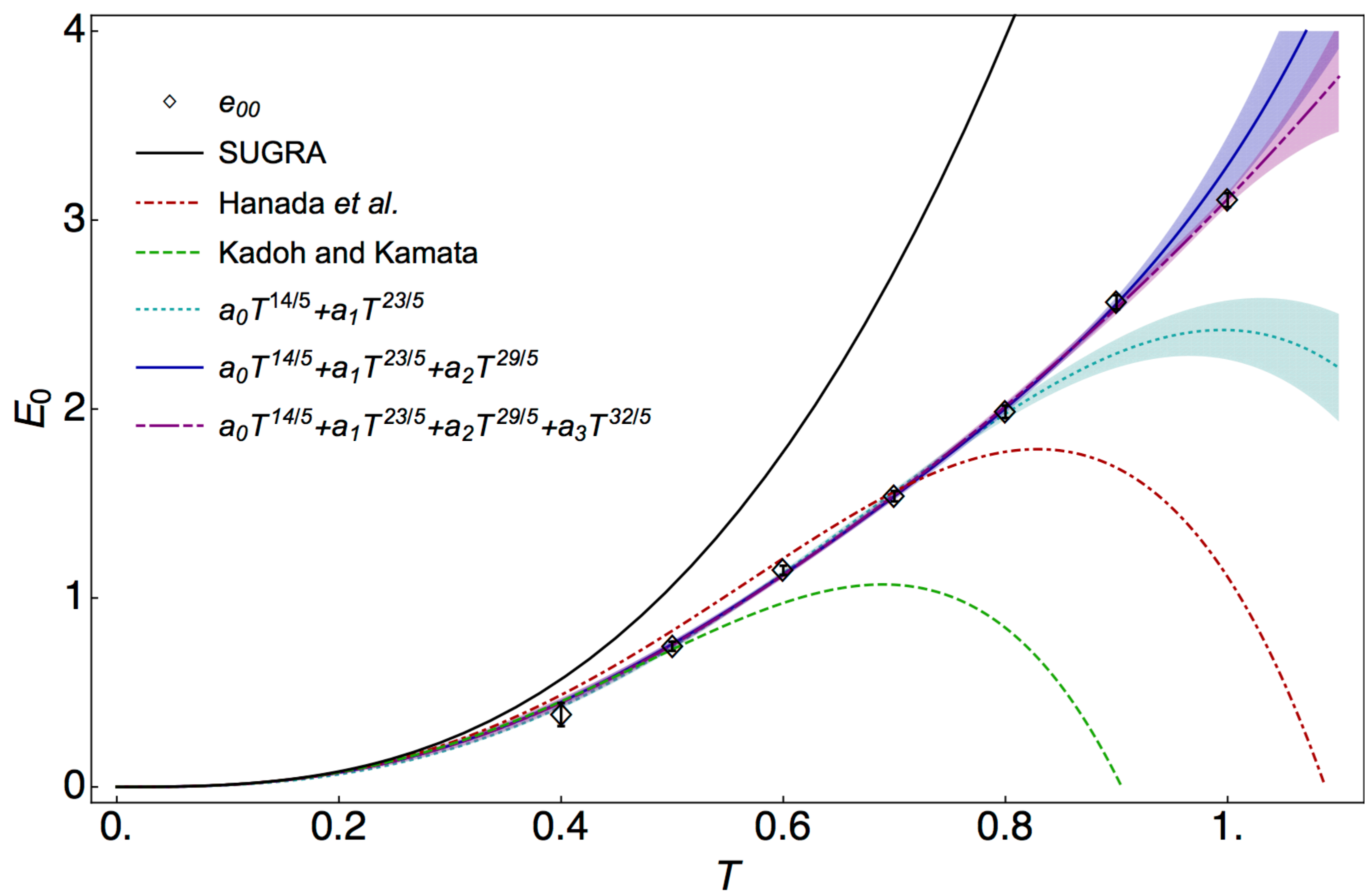
State-of-the-art lattice results

- $a_0 = 7.41$ is known from SUGRA calculations.
- Finite-T corrections => α' corrections in Type II string theory

The coefficients a_1, a_2 unknown from supergravity. We only know that corrections start at … $(\alpha')^3$

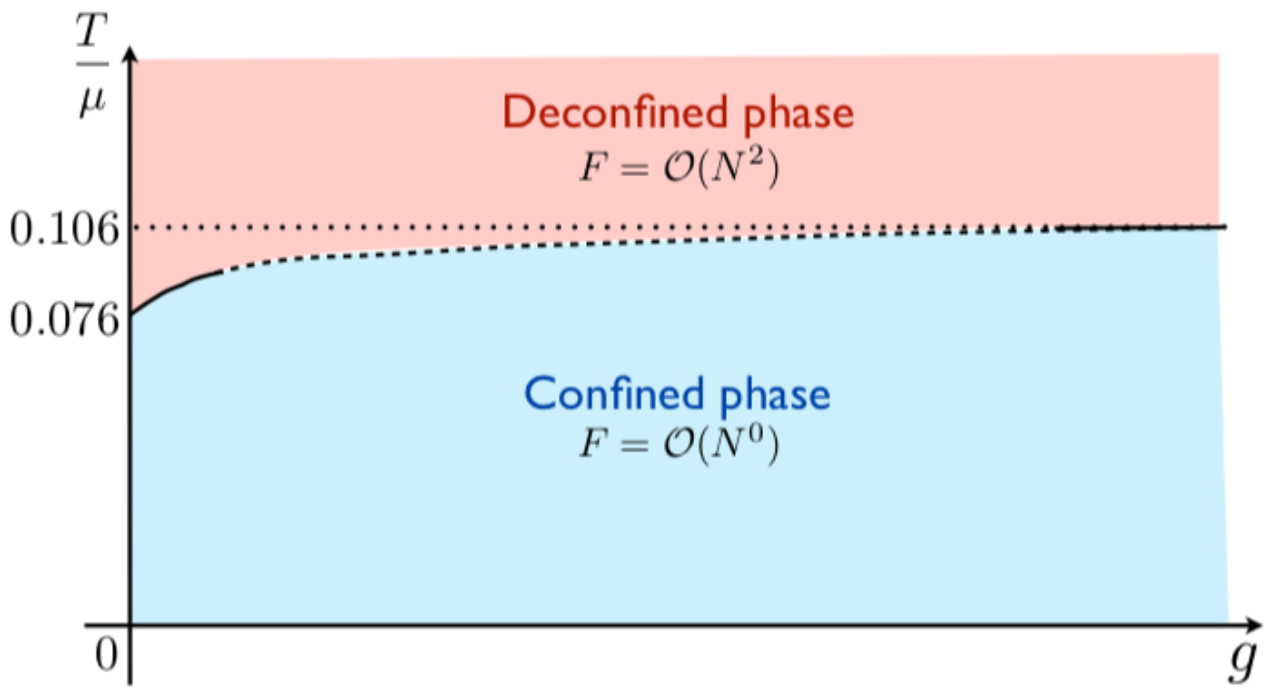
Figure from arXiv: 1606.04951

(M. Hanada, G. Ishiki et al.)



Phase transition?

Can we study some holographic model with some interesting phase structure ? BFSS matrix model only has a *deconfined* phase. However, when we consider the massive deformation of this model (known as BMN or PWMM) model, there is an interesting phase structure.



1411.5541 (Costa, Penedones, Greenspan, Santos)

BMN matrix model

$$S_{BMN} = S_{BFSS} - \frac{N}{4\lambda}\int d\tau \, \mathrm{Tr}\left(\frac{\mu^2}{3^2}\left(X^I\right)^2 + \frac{\mu^2}{6^2}\left(X^M\right)^2 + \frac{2\mu}{3}\epsilon_{IJK}X^I X^J X^K + \frac{\mu}{4}\overline{\Psi}^{\alpha}\left(\gamma^{123}\right)_{\alpha\beta}\Psi^{\beta} \right)$$

The flat directions of the BFSS model are lifted by giving masses to SO(3) and SO(6) scalars. In addition, there is a cubic scalar term which is also known as 'Myers term' plus a fermion term. Unlike BFSS, this model is on pp-wave spacetime. Even after the addition of these mass terms, supersymmetry is intact!

Dual classical gravity solution valid when,

$$g = \lambda/\mu^3 \gg 1 \text{ with } \mu \ll 1 \text{ and } N \to \infty$$

BMN phase diagram

(S. Catterall, A. Joseph, D. Schaich, RGJ, T.Wiseman, 19XX.XXXXX)

Use naive discretisation to study BMN on the lattice. Since, these theories is super renormalizable in d < 4, such a discretisation is still reasonable for 1d SUSY theories.

The order parameter for the thermal (deconfinement) transition is Polyakov loop. Finite volume phase transition in the large N limit.

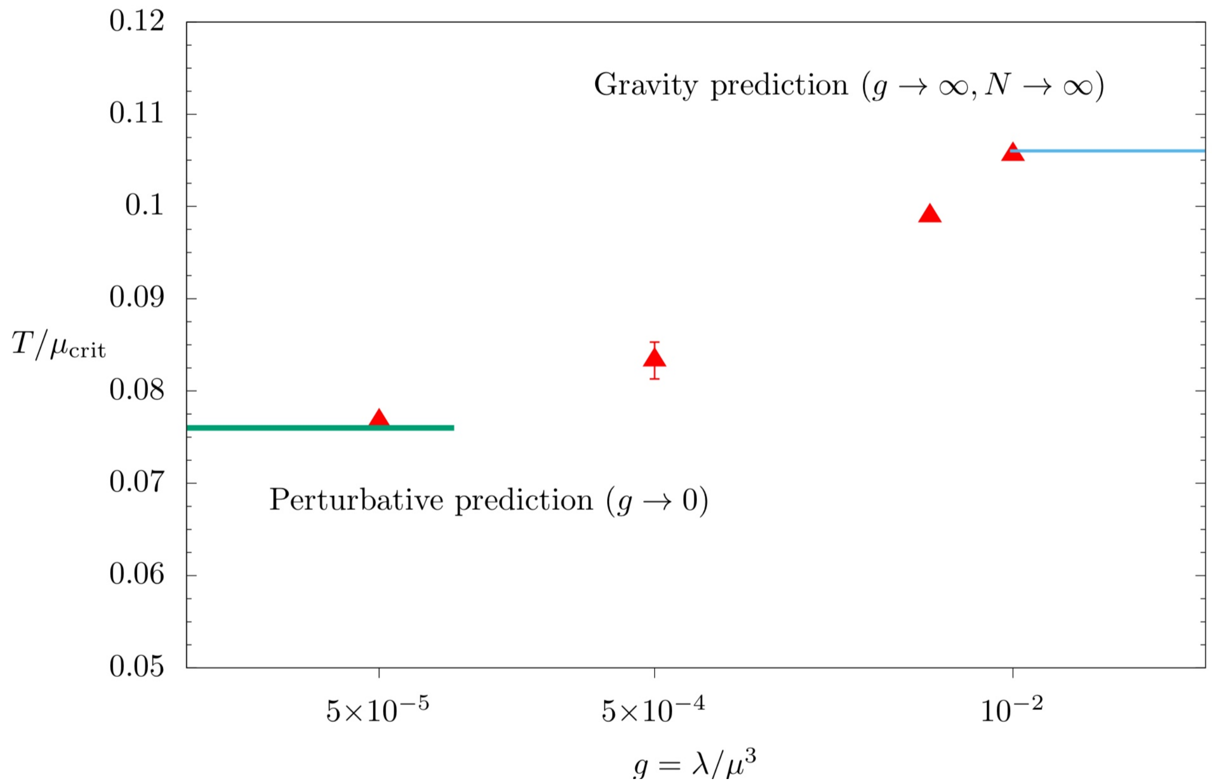
In fact, another possible order parameter for these finite-T transitions is Entanglement Entropy (EE).

$$\langle P \rangle \neq 0 \;\; \text{Deconfined} \qquad\qquad \langle P \rangle = 0 \;\; \text{Confined}$$

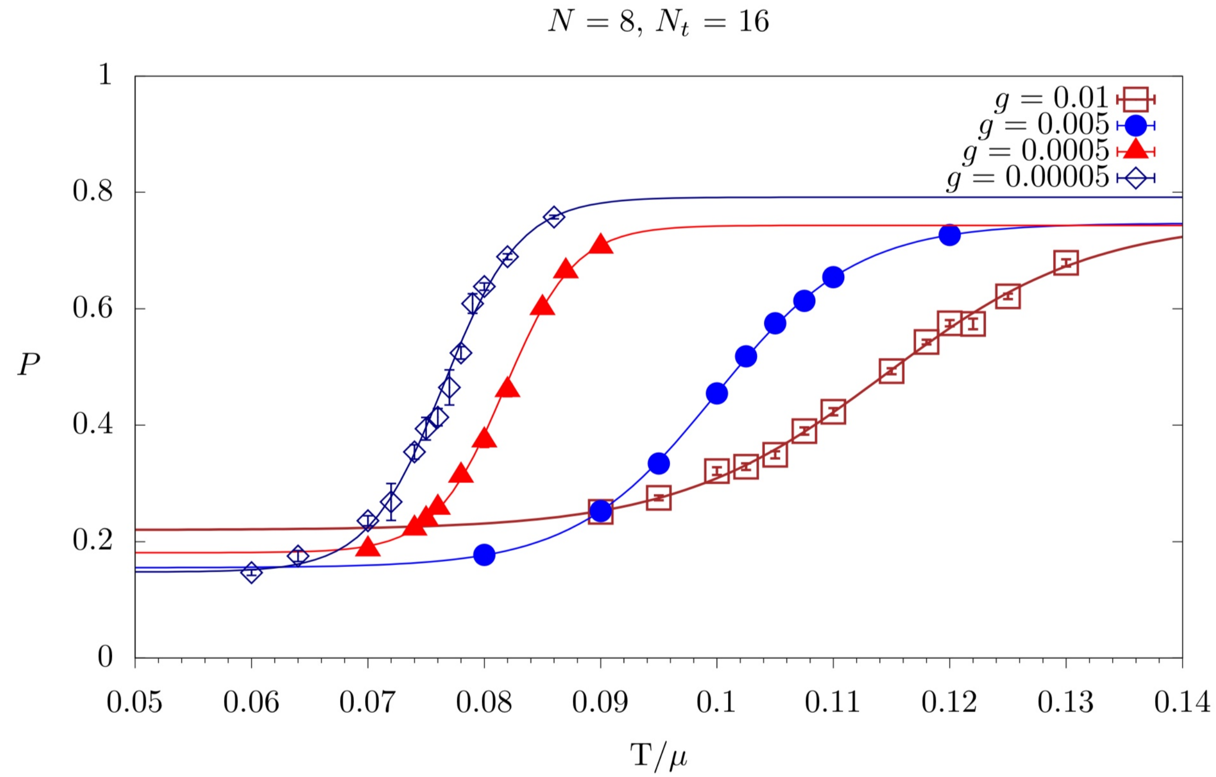
First explore, $g \to 0$, and see if the perturbative results (large N) are reproduced.

BMN phase diagram … continued

(S. Catterall, A. Joseph, D. Schaich, RGJ, T.Wiseman, 19XX.XXXXX)



Finite coupling phase transition from lattice calculations.



Polyakov loop as order parameter

Ongoing & future work in 3d SYM and N=4 SYM

- Trying to understand the thermodynamics of dual uniform D2-branes from 3d SYM at strong coupling and large N (in preparation).
- Calculate the static quark potential and check predictions by Maldacena (**hep-th/9803002**).
- Well-known dependence of V ~ $\sqrt{\lambda}$ in 4d. Generally, it is expected to follow $\lambda^{1/(5-p)}$ for a (p+1)-dim SYM theory in regimes where supergravity (SUGRA) description is valid.

Conclusion

Studying holography using numerical methods can refine our understanding of quantum gravity by directly dealing with first principle studies of strongly coupled systems with/without supersymmetry and statistical systems at criticality.

Thank you!