Testing holographic principle using lattice simulations

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Overview

1. Motivation
2. Lattice construction of $\mathcal{N}=4$ SYM
3. Gauge/Gravity duality
4. Results

Motivation

Study maximally supersymmetric Yang-Mills (SYM) in $(p + 1)$ dimensions for $p < 3$. This is conjectured to be dual to Type IIA/B superstring theory containing stack of N Dp-branes in the *decoupling limit*.

- ⇒ At low temperatures (strong coupling), there is a dual supergravity theory (as low-energy description of Type II string theory).
- ⇒ We want to use gauge/gravity duality to understand it from SYM theory.
- ⇒ In this case, the gauge theory is strongly coupled and we use *lattice* to study this system.

In this talk, I will focus on the p=1 case

Lattice construction of $\mathcal{N}$=4 SYM

Supersymmetry extends Poincaré symmetry adding spinorial generators Q and $\overline{Q}$ to translations, rotations, boosts

The algebra includes $\{Q, \overline{Q}\} \sim P_\mu$

SUSY on the lattice

Supersymmetry extends Poincaré symmetry adding spinorial generators Q and $\overline{Q}$ to translations, rotations, boosts

The algebra includes $\{Q, \overline{Q}\} \sim P_\mu$

Fortunately, there are certain constructions where we can exactly preserve a subset of SUSY algebra on the lattice based on *twisted* and orbifold constructions. See [0903.4881] for review.

Requirements : Enough supercharges in the continuum (at least 2^D, where D is space-time dimensions)

Unique features :

- ⇒ Single supercharge exactly preserved on the lattice in four dimensions.
- ⇒ Gauge symmetry, $\mathbb{Q}$ nilpotent symmetry, S_5 point group symmetry.

$\mathcal{N} = 4$ SYM

—The only known 4d theory with a supersymmetric lattice formulation. Also the simplest non-trivial field theory in four dimensions without gravity.

—Context for development of AdS/CFT correspondence in large-N limit at strong couplings

SU(N) gauge theory with four fermions Ψ^{I} and six scalars Φ^{IJ}, all massless and in adjoint rep.

Supersymmetric: 16 supercharges Q^{I}_{α} and $\overline{Q}^{\mathrm{I}}_{\dot{\alpha}}$ with $\mathrm{I} = 1, \cdots, 4$. Fields and Q's transform under global SU(4) $\simeq$ SO(6) R-symmetry

Conformal: β function is zero for any 't Hooft coupling $\lambda = g^2_{YM} N$

Twisted construction

$$SO(4)_{tw} \equiv \text{diag}\left[SO(4)_{euc} \times SO(4)_R\right] \quad ; \quad SO(4)_R \subset SO(6)_R$$

The 16-real components of the spinors in $\mathcal{N} = 4$ SYM fill up the Dirac-Kähler multiplet :

$$\begin{pmatrix} Q^1_\alpha & Q^2_\alpha & Q^3_\alpha & Q^4_\alpha \\ \overline{Q}^1_{\dot\alpha} & \overline{Q}^2_{\dot\alpha} & \overline{Q}^3_{\dot\alpha} & \overline{Q}^4_{\dot\alpha} \end{pmatrix} = \mathbb{Q} + \gamma_\mu \mathbb{Q}_\mu + \gamma_\mu\gamma_\nu \mathbb{Q}_{\mu\nu} + \gamma_\mu\gamma_5 \mathbb{Q}_{\mu\nu\rho} + \gamma_5 \mathbb{Q}_{\mu\nu\rho\sigma}$$
$$\longrightarrow \mathbb{Q} + \gamma_a \mathbb{Q}_a + \gamma_a\gamma_b \mathbb{Q}_{ab}$$
$$\text{with } a, b = 1, \cdots, 5$$

$\mathbb{Q}$'s transform with integer spin under the "twisted rotation group".

Twisting and repackaging gives a nilpotent, scalar supercharge $\mathbb{Q}$ which can be exactly preserved on the lattice.

Twisted $\mathcal{N} = 4$ SYM fields

Start from a 5d setup

$$Q \text{ and } \overline{Q} \longrightarrow \mathbb{Q} + \mathbb{Q}_a + \mathbb{Q}_{ab}$$

$$\Psi \text{ and } \overline{\Psi} \longrightarrow \eta,\ \psi_a \text{ and } \chi_{ab}$$

$$A \text{ and } \Phi \longrightarrow \mathcal{A}_a \text{ and } \overline{\mathcal{A}}_a$$

Everything transforms with **integer spin** under $SO(4)_{tw}$ — **no spinors**. Then under dimensional reduction :

$$\mathbb{Q},\ \mathbb{Q}_a \text{ and } \mathbb{Q}_{ab} \longrightarrow \mathbb{Q} + \gamma_\mu \mathbb{Q}_\mu + \gamma_\mu \gamma_\nu \mathbb{Q}_{\mu\nu} + \gamma_\mu \gamma_5 \mathbb{Q}_{\mu\nu\rho} + \gamma_5 \mathbb{Q}_{\mu\nu\rho\sigma}$$

$$\mathcal{A}_a \longrightarrow (A_\mu, \phi) + i(\Phi_\mu, \overline{\phi})$$

where, a b runs from 1 ·· 5 and μ from 1 ·· 4

Public code on GitHub

Code for supersymmetric construction of $\mathcal{N} = 4$ SYM evolved from MILC lattice QCD code and is hosted on GitHub.

Download, Fork, Contribute

https://github.com/daschaich/susy

Gauge/Gravity duality

Duality (one-way)

Gravitational theory

Weakly coupled (low energy) string theory

Stack of N Dp-branes, N units of charge at temperature T

Gauge theory

16 supercharge SYM theory in D = p+1 dimensions

SU(N) gauge group with large N, strongly coupled at temperature T

The map for p=1

- ⇒ Gravity has two different phases : Homogeneous black string & localized black hole with a first-order phase transition between them.
- ⇒ Gauge theory *should* have a deconfinement phase transition where deconfined phase is dual to - localized black hole phase *and* confined phase is dual to - homogeneous black string. Both phases have *different* thermodynamic behavior. Valid only at strong coupling and large-N.
- ⇒ <u>Our aim</u> : Confirm that the *map* is consistent through lattice calculations.

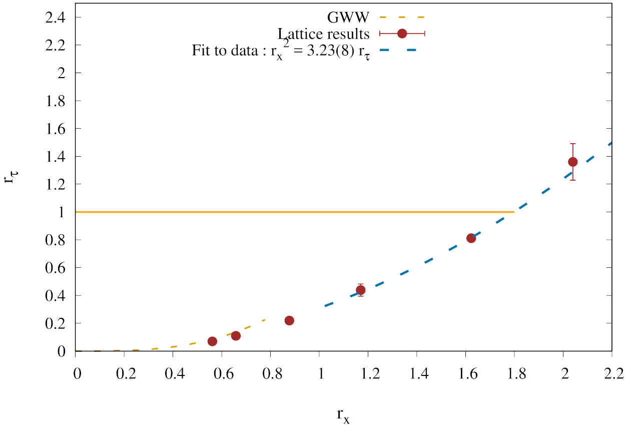
2d SYM - setting up

- ⇒ Dimensionally reduce the 4d theory to two dimensions.
- ⇒ We can construct dimensionless extents in two directions as : $r_{\rm x} = \sqrt{\lambda}L$, $\mathrm{r}_\tau = \sqrt{\lambda}\beta$. Dimensionless temperature, $t = 1/r_\tau$. Strong couplings implies low temperatures.
- ⇒ At high temperatures and $r_{\rm x} \gg r_\tau$, there is a third (GWW) and closely separated second order phase transition. When coupling is increased to $r_\tau \gg 1$ and $\alpha = r_{\rm x}/r_\tau = TL \approx \mathcal{O}(1)$, the gravity description kicks in and we have a first-order phase transition.

⇒ Gravity predicts the transition to occur across : $\mathrm{r}_x^2 = c_{\rm grav} r_\tau$.

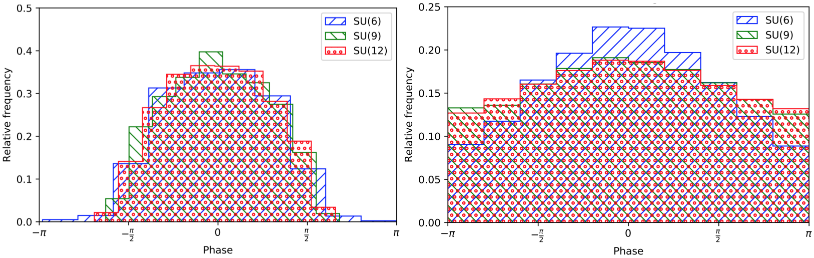
Results

Deconfinement transition

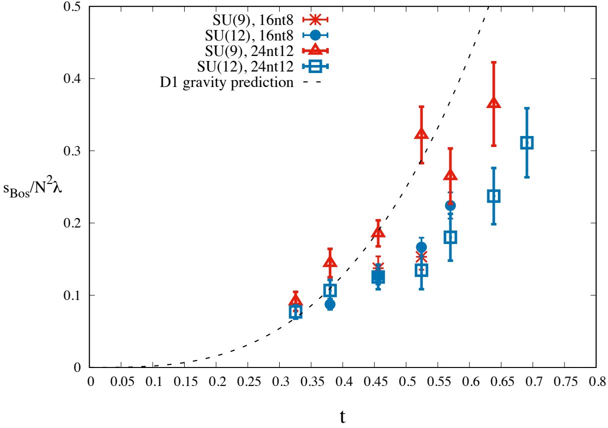


$r_x^2 > c_{\text{grav}} r_\tau$ corresponds to the homogeneous phase. The order parameter for the phase transition is the Wilson line.

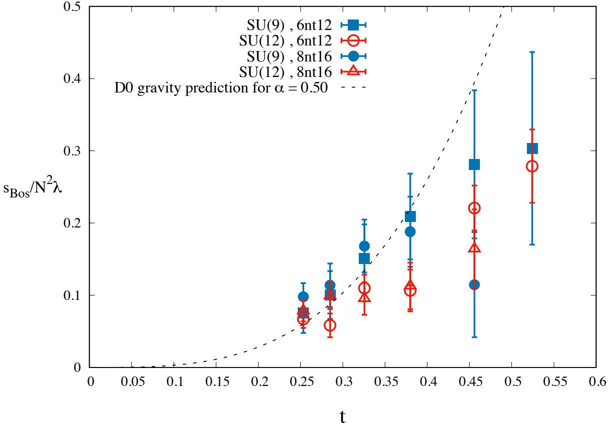
Unitarized Wilson line phases - localized and uniform



D1 gravity (homogeneous phase) - thermodynamics



D0 gravity (localized phase) - thermodynamics



Conclusions

⇒ Sixteen supercharge SYM theory is now possible to study at large N using twisted lattice construction in various dimensions.

⇒ Positive evidence from lattice simulations of strongly coupled SYM theory at large N that gauge/gravity duality might be correct.

Thank you.

Funding and computing resources

Other details (if needed)

⇒ No sign problem with anti-periodic boundary conditions for fermions (which we use here).

⇒ The U(1) mode is truncated from the start, but, restored at sufficiently large N. See talk by Joel Giedt @ Lattice 2017.

⇒ To regulate SU(N) flat directions, we added a small mass term μ. We extrapolated the energy density to the $\mu \to 0$ limit.

⇒ Soft-mass term added to ensure that center symmetry is completely broken along reduced directions.

⇒ The breaking of supersymmetry is within few % with the largest N we simulate.