Supersymmetry breaking and gauge/gravity duality on the lattice

Raghav G. Jha
(Final Year PhD student)

at Syracuse University

1709.07025 [PRD, in press] , 1800.00012 [PRD **97**, 054504] and work in progress with Simon Catterall, Joel Giedt, Anosh Joseph, David Schaich & Toby Wiseman

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Outline

- Motivation and possibilities
- Two dimensional $\mathcal{N} = (2,2)$ SYM – supersymmetry breaking
- Holographic connection - two and three dimensional SYM (16 supercharges)

Why lattice supersymmetry (SUSY) ?

Discretization on the lattice furnishes gauge-invariant regularization of gauge theories and provides non-perturbative insights into

- Gauge/gravity (AdS/CFT) duality - potential non-perturbative definition of string theory
- Finite N regime and large N limit of supersymmetric theories.
- Confinement, phase transitions, symmetry breaking and conformal field theories.

Lattice SUSY : Problem and resolution

Problem

Supersymmetry generalizes Poincaré symmetry by adding spinorial generators Q and $\bar{Q}$ to translations, rotations, boosts

The algebra includes $Q\bar{Q} + \bar{Q}Q = 2\sigma^{\mu} P_{\mu}$,

P_{μ} generates infinitesimal translations, which don't exist on the lattice. Supersymmetry explicitly broken at the classical level.

Solution

Preserve a subset of SUSY algebra exactly on the lattice. Possible for theories with $Q \geq 2^{D}$. For ex : $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM). Methods are based on orbifold construction and topological twisting. I will focus only on the twisted construction in this talk.

Lattices in various dimensions

THEORY	R-SYMMETRY	LATTICE CONSTRUCTION ?
$d = 2, \mathcal{Q} = 4$	$SO(2) \bigotimes U(1)$	✓
$d = 2, \mathcal{Q} = 8$	$SO(4) \bigotimes SU(2)$	✓
$d = 2, \mathcal{Q} = 16$	$SO(8)$	✓
$d = 3, \mathcal{Q} = 4$	$U(1)$	
$d = 3, \mathcal{Q} = 8$	$SO(3) \bigotimes SU(2)$	✓
$d = 3, \mathcal{Q} = 16$	$SO(7)$	✓
$d = 4, \mathcal{Q} = 4$	$U(1)$	
$d = 4, \mathcal{Q} = 8$	$SO(2) \bigotimes SU(2)$	
$d = 4, \mathcal{Q} = 16$	$SO(6)$	✓

SUSY breaking : Witten index

To understand susy breaking non-perturbatively, Witten introduced index, W. As it turns out, W can be written as,

$$W = \int_{PBC} \mathscr{D}(\cdots)e^{-S}$$

don't have a way of evaluating this using simulations

Alternatively : Look for ground state energy as order parameter for breaking

$\mathcal{N} = (2,2)$ SYM in d=2

The action of continuum $\mathcal{N} = (2,2)$ SYM takes the following $\mathcal{Q}$-exact form after topological twisting

$$S = \frac{N}{2\lambda} \mathcal{Q} \int d^2x \, \Lambda,$$

where

$$\Lambda = \mathrm{Tr} \, \left(\chi_{\mu\nu} \mathcal{F}_{\mu\nu} + \eta [\overline{\mathcal{D}}_\mu, \mathcal{D}_\mu] - \frac{1}{2} \eta d \right),$$

and $\lambda = g^2 N$ is the 't Hooft coupling.

The nilpotent supersymmetry transformations associated with the scalar supercharge $\mathcal{Q}$ are given by

$$
\begin{aligned}
\mathcal{Q}\, \mathcal{A}_\mu &= \psi_\mu, \\
\mathcal{Q}\, \psi_\mu &= 0, \\
\mathcal{Q}\, \overline{\mathcal{A}}_\mu &= 0, \\
\mathcal{Q}\, \chi_{\mu\nu} &= -\overline{\mathcal{F}}_{\mu\nu}, \\
\mathcal{Q}\, \eta &= d, \\
\mathcal{Q}\, d &= 0.
\end{aligned}
$$

The four degrees of freedom appearing in this theory are just the twisted fermions $(\eta, \psi_\mu, \chi_{\mu\nu})$ and complexified gauge field $\mathcal{A}_\mu$. The complexified field is constructed from the usual gauge field A_μ and the two scalars B_μ present in the untwisted theory: $\mathcal{A}_\mu = A_\mu + iB_\mu$. The twisted theory is naturally written in terms of the complexified covariant derivatives

$$\mathcal{D}_\mu = \partial_\mu + \mathcal{A}_\mu, \qquad \overline{\mathcal{D}}_\mu = \partial_\mu + \overline{\mathcal{A}}_\mu, \tag{1}$$

and complexified field strengths

$$\mathcal{F}_{\mu\nu} = [\mathcal{D}_\mu, \mathcal{D}_\nu], \qquad \overline{\mathcal{F}}_{\mu\nu} = [\overline{\mathcal{D}}_\mu, \overline{\mathcal{D}}_\nu]. \tag{2}$$

The action can be written as, $\mathrm{S} = \mathrm{S}_B + S_F$,where the bosonic action is

$$S_B \;=\; \frac{N}{2\lambda}\sum_{\mathbf{n}} \mathrm{Tr}\,\Big(-\overline{\mathcal{F}}_{\mu\nu}(\mathbf{n})\mathcal{F}_{\mu\nu}(\mathbf{n}) + \frac{1}{2}[\overline{\mathcal{D}}_{\mu}, \mathcal{D}_{\mu}]^2\Big),$$

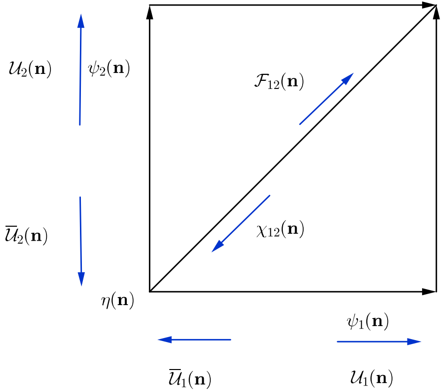
and the fermionic piece

$$S_F \;=\; \frac{N}{2\lambda}\sum_{\mathbf{n}} \mathrm{Tr}\,\Big(-\chi_{\mu\nu}(\mathbf{n})\mathcal{D}_{[\mu}\psi_{\nu]}(\mathbf{n}) - \eta(\mathbf{n})\overline{\mathcal{D}}_{\mu}\psi_{\nu}(\mathbf{n})\Big).$$

Also an additional mass term (breaks $\mathcal{Q}$ supersymmetry)

$$S_{\mathrm{soft}} = \frac{N}{2\lambda}\mu^2 \sum_{\mathbf{n},\mu} \mathrm{Tr}\,\Big(\overline{\mathcal{U}}_{\mu}(\mathbf{n})\mathcal{U}_{\mu}(\mathbf{n}) - \mathbb{I}_N\Big)^2,$$

Fields on the lattice



Extrapolations [PRD **97**, 054504]

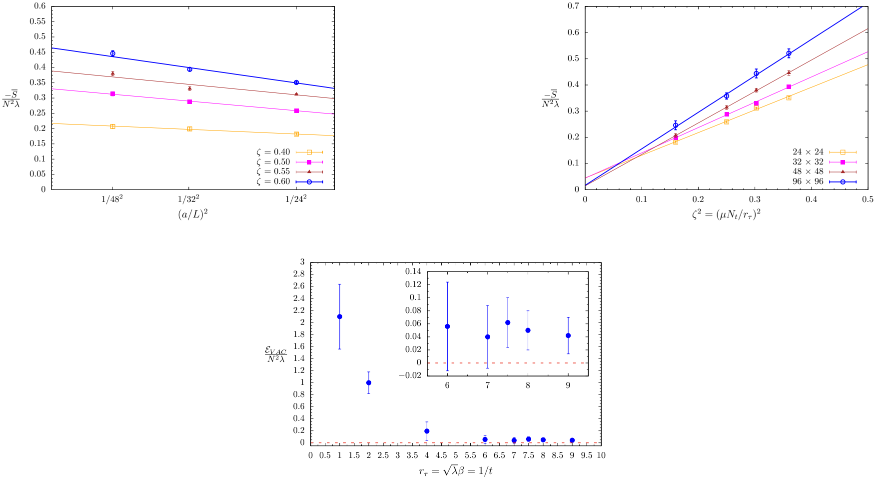


Figure: Left : $\lim_{\mathrm{a}\to 0}$, Right : $\lim_{\mu^2\to 0}$, Bottom : $\lim_{\beta\to\infty}$

Supersymmetry breaking

- Calculate the ground state energy density in the limit $\beta \to \infty$? (why not just do T$=0$ calculation)
- Need to use small mass term μ to control flat directions, which we extrapolate to zero after doing continuum extrapolation (a $\to 0$).
- Upper bound on energy density $\frac{\mathcal{E}_{\text{VAC}}}{N^2\lambda} = 0.05(2)$, statistically consistent with zero.

[Similar study done earlier by Kanamori, Sugino and Suzuki based on A-twist Sugino's action]

Applications to holography - gauge/gravity

Original AdS/CFT correspondence

4D $\mathcal{N} = 4$ U(N) super-Yang-Mills theory associated with N D3-branes, is dual to Type IIB string theory on $AdS_5 \times S_5$ in the large N limit.

More general holographic dualities in lower dimensions

Maximally supersymmetric YM in $p+1$ dimensions dual to Dp-branes

At low temperatures, and in the decoupling limit : dual description in terms of black holes in Type II A/B supergravity

Decoupling limit: $N \to \infty$ and $t = T/\lambda^{\frac{1}{3-p}} \ll 1$

Maximal SYM for p < 3

- Dimensionally reduce lattice $\mathcal{N} = 4$ SYM along (3-p) spatial directions.
- Dimensional reduction : $A_4^* \rightarrow$ A_{p+1}^* giving a skewed torus with $\gamma = -1/(p+1)$ ($\gamma = \cos\theta$).
- 't Hooft coupling (λ) is dimensionful in p < 3 dimensions and we construct a dimensionless coupling given by $\mathrm{r}_{\text{eff}} = \lambda_p \beta^{3-p}$, where $\beta = 1/T$.
- No phase transition (single de-confined phase) in 1-d QM case, richer structure for p = 1,2.

Regime of valid supergravity (SUGRA) description

To have a valid SUGRA description, we need :

- Radius of curvature should be large in units of α'. This implies $r_{\text{eff}} \gg 1$.
- String coupling should be small.

We can combine both requirements to get a constraint on the effective dimensionless coupling we can probe for a well-defined SUGRA description (p $<$ 3)

$$1 \ll \lambda_p \beta^{3-p} \ll N^{\frac{10-2p}{7-p}}$$

Various dimensions - Existing works

- p=0 : [Hanada, Nishimura and Takeuchi in 0706.1647 + Catterall & Wiseman, 0706.3518]
- p=1 : This talk [Our recent work arXiv: 1709.07025 (PRD, in press), also work done using different action by D. Kadoh.]
- p=2 : This talk [Preliminary work]

Eventual goal, p=3 : Thermodynamics of $\mathcal{N} = 4$ SYM. Statement : Can we understand $\mathrm{f}(\lambda) \ni$, $\mathrm{f}(0) = 1$ and $\mathrm{f}(\infty) = 3/4$?

p=1 : Maximal SYM in (1+1)-dimensions

- Interesting phase structure at finite temperature with a deconfinement transition dual to a gravity transition (between uniform D1 and localized D0 phase with spatial Wilson loop being the order parameter) at strong coupling and large N.
- Different temperature dependence in both phases for free energy (D0 & D1 thermodynamics)
- Can see the transition but can't determine the order !

Some results (arXiv: 1709.07025)

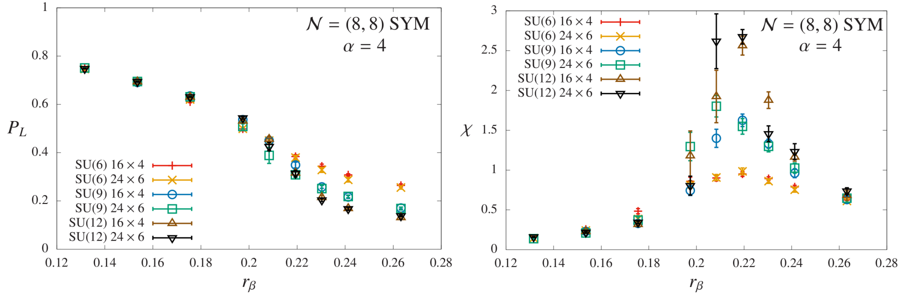


Figure: Spatial Wilson loop magnitude (left) and susceptibility (right) vs. inverse dimensionless temperature $r_\beta = 1/t$ for SU(N) gauge groups with $N = 6$, 9 and 12 on 16×4 and 24×6 lattices (aspect ratio $\alpha = N_x/N_t = 4$). The transition strengthens as N increases, while showing little sensitivity to the lattice size.

Some results (arXiv: 1709.07025)

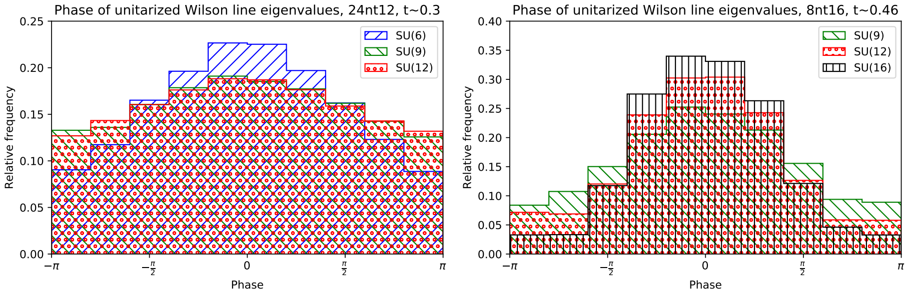
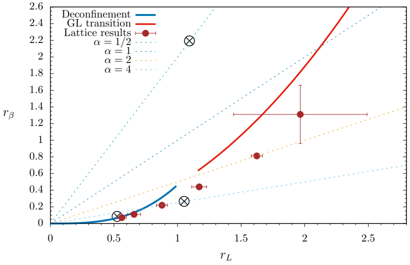


Figure: Wilson line phases. Uniform distribution (left) and localized distribution (right) corresponding to different black hole phases in the dual theory.



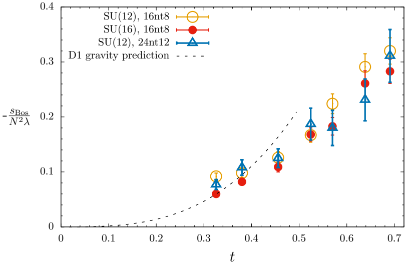
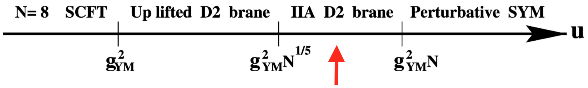


Figure: The critical temperature for different α-lattices (left). D1 phase thermodynamics for $\alpha = 2$ (right). Dashed curve (right) is gravity prediction.

p=2 : Maximal SYM in (2+1)-dimensions

’t Hooft coupling has dimensions of energy. Construct $r_{\text{eff}} = \lambda\beta = 1/t$ as dimensionless coupling. Type IIA SUGRA description is valid when the energy scale, $u = r/\alpha'$ (defined as fixed expectation value of a scalar) is in the range shown below :

O. Aharony et al. / Physics Reports 323 (2000) 183–386



This translates to the condition (for our dimensionless coupling) as,

$$1 \ll r_{\text{eff}} \ll N^{\frac{6}{5}}$$

Divergence of thermal partition function - I

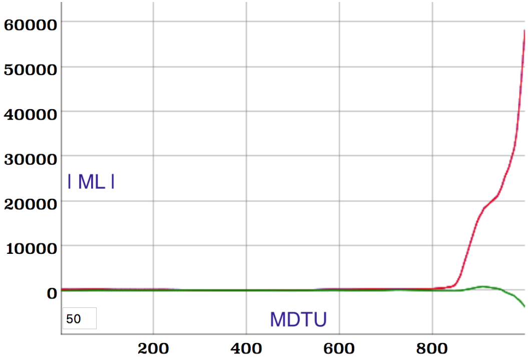
First discussed by [Kabat, Lifshitz and Lowe, hep-th/9910001, hep-th/0105171], the thermal SYM partition function has divergence.

$$I \sim kN \log(f(\zeta)) + N^2 I_{\text{finite}}$$

So technically, one can avoid the issue of divergence if $N \to \infty$ (another need for large N) because the finite contribution dominates. For the N we can access in our numerical simulations, we need to do more !

Use a mass term for the scalar fields in our lattice action to restrict the moduli space and then extract the finite piece carefully and compare to the thermodynamics of Dp-branes.

An example of divergence showing up at N=4.



Thermodynamics of D2-branes

For a uniform Dp-brane (p < 3) , we have a prediction for free energy density which is [Itzhaki et al., hep-th/9802042, Harmark and Obers, hep-th/0407094],

$$\mathcal{F} = -k_p N^2 \lambda^{\frac{1+p}{3-p}} t^{\frac{14-2p}{5-p}}$$

where, k can be read off the table in the above reference.

p	0	1	2	3	4
k_p	$(2^{21}3^2 5^7 7^{-19}\pi^{14})^{1/5}$	$2^4 3^{-4}\pi^{5/2}$	$(2^{13}3^5 5^{-13}\pi^8)^{1/3}$	$2^{-3}\pi^2$	$2^5 3^{-7}\pi^2$

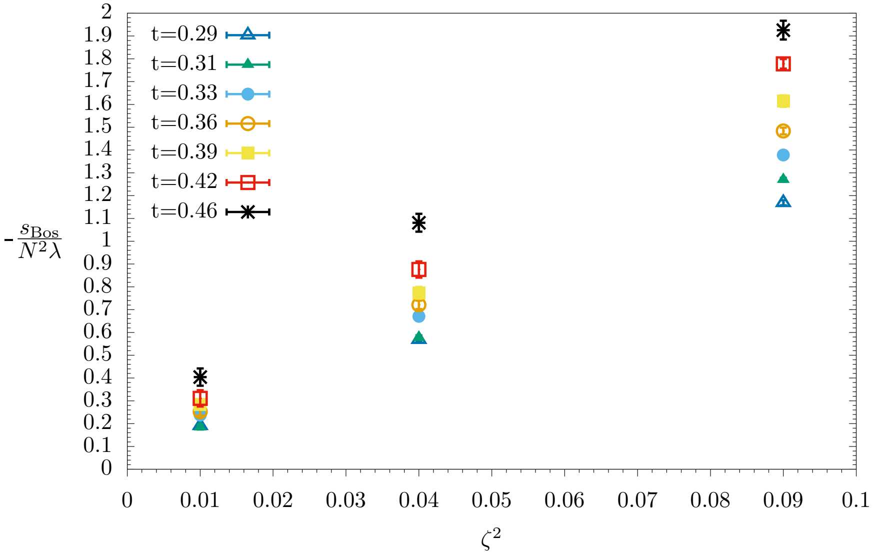
For our case of *i.e* p = 2, we get :

$$\mathcal{F} = -2.492\ N^2 \lambda^3 t^{\frac{10}{3}}$$

Lattice simulations

- We focus on calculating the free energy density for the SYM theory on the lattice restricting to uniform D2 phase.
- Choose temperatures $t \ll 1$ and large N for multiple lattices.
- Computational cost scales as $\sim N^{7/2}$, so we restrict to $N_{\text{maximum}} = 8$ on $8^3, 10^3$ and 12^3 lattices.
- We need to use small mass regulator ζ (discussed before), which we extrapolate to zero as $\zeta^2 \to 0$.
- Publicly available lattice code for arbitrary N (we have explored up to N=20 with fermions in 1d, unpublished) : github.com/daschaich/susy

Preliminary numerical results



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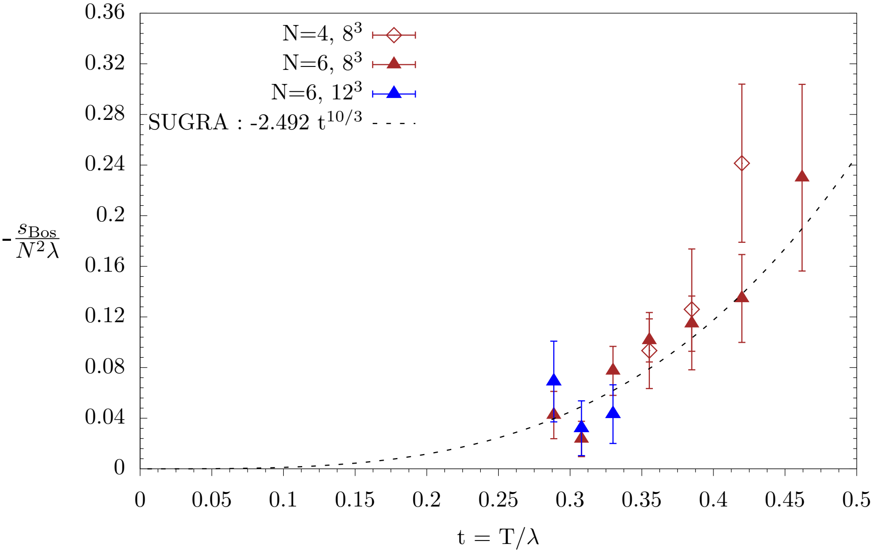


Thank you !

Thank you !

Funding and computing resources

Truncated lattice theory

A naïve truncation of U(N) supersymmetric theory to SU(N) does not work at finite-N.

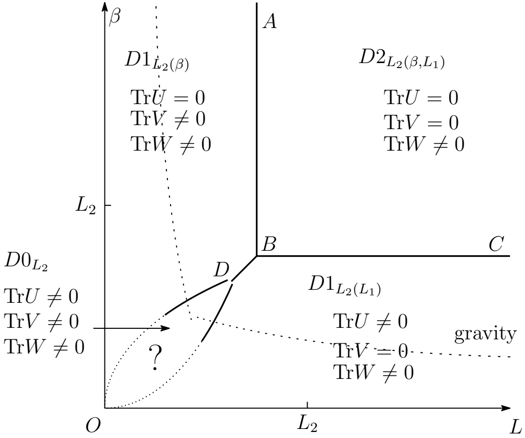
- Breaks the lattice supersymmetry that relates $\mathcal{U}_a$ to ψ_a in the U(N) construction.
- Solution : Represent the truncated gauge links as $\mathcal{U}_b = e^{iga\mathcal{A}_b}$ to argue that the continuum supersymmetry relating $\mathcal{A}_a$ and ψ_a is approximately realized in the large-N limit even at non-zero lattice spacing since $g \to 0$ in the decoupling limit.

Continuum vs. lattice coupling

The non-orthogonal basis vectors of the A_d^* lattice leads to mismatch in 't Hooft coupling between lattice and continuum. The target continuum (p+1)-SYM coupling ($r_{\tau,cont.}$) differs from the lattice coupling as,

$$r_{\tau,\text{lattice}} = \frac{(d+1)^{\frac{4-p}{6-2p}}}{\sqrt{d}} r_{\tau,\text{cont}}$$

Backup 1



β
A
$D1_{L_2(\beta)}$
$\mathrm{Tr}U = 0$
$\mathrm{Tr}V \neq 0$
$\mathrm{Tr}W \neq 0$
$D2_{L_2(\beta,L_1)}$
$\mathrm{Tr}U = 0$
$\mathrm{Tr}V = 0$
$\mathrm{Tr}W \neq 0$
L_2
B
C
$D0_{L_2}$
$\mathrm{Tr}U \neq 0$
$\mathrm{Tr}V \neq 0$
$\mathrm{Tr}W \neq 0$
D
$D1_{L_2(L_1)}$
$\mathrm{Tr}U \neq 0$
$\mathrm{Tr}V = 0$
$\mathrm{Tr}W \neq 0$
gravity
?
O
L_2
L

Backup 2

Lower-dimensional sixteen supercharge SYM with **apbc** has no sign problem.