HOLOGRAPHY, LARGE N, AND SUPERSYMMETRY ON THE LATTICE

Raghav Govind Jha
April 2, MMXIX

The month of April!

- April 8, 2016 – Syracuse, NY   (Mid Ph.D. defense)
- April 4, 2017 – Kyoto, Japan (Simon’s Invited talk)
- April 6, 2018 – Boulder, CO   (Invited talk)
- April 2, 2019 – Today

Not included

1. Dynamical supersymmetry breaking in 1+1-SYM four-supercharge SYM *(arXiv 1801.00012)*

2. Hydrodynamical properties of D1 branes from lattice simulations of SYM theory in 1+1-dimensions using gauge/gravity duality **(arXiv 1809.00797)**

3. Lattice quantum gravity with scalar fields **(arXiv 1810.09946)**

4. Tensor renormalization group study of 2d non-Abelian gauge Higgs model **(arXiv 1901.11143)**

Sketch

1. Introduction to Gauge/gravity duality

2. Lattice construction of $\mathcal{N} = 4$ super Yang-Mills (SYM)

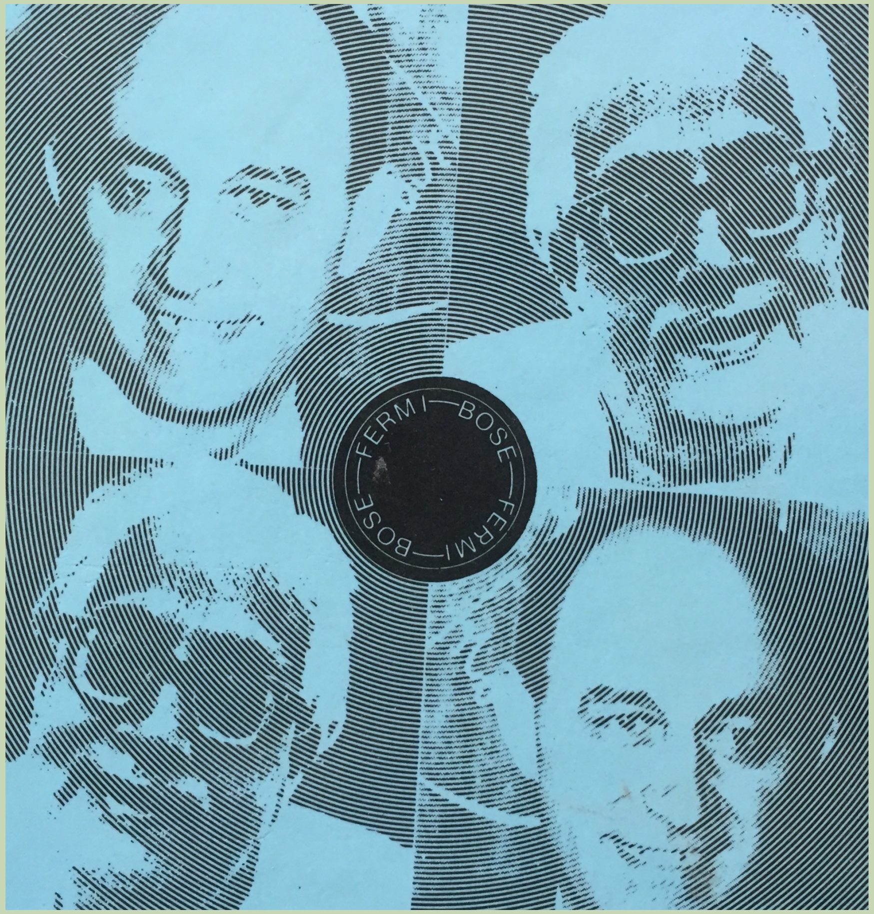
3. Dimensional reductions of $\mathcal{N} = 4$ SYM

4. Holographic dual gauge theories (all large N)***** – BFSS matrix model, BMN matrix model, 1+1-SYM, 2+1 SYM, 4d $\mathcal{N}$=4 SYM (**arXiv 1709.07025, 1808.04735, 19XX. XXXXX, 19XX. XXXXX**)

5. Future directions + Conclusions

What is supersymmetry?

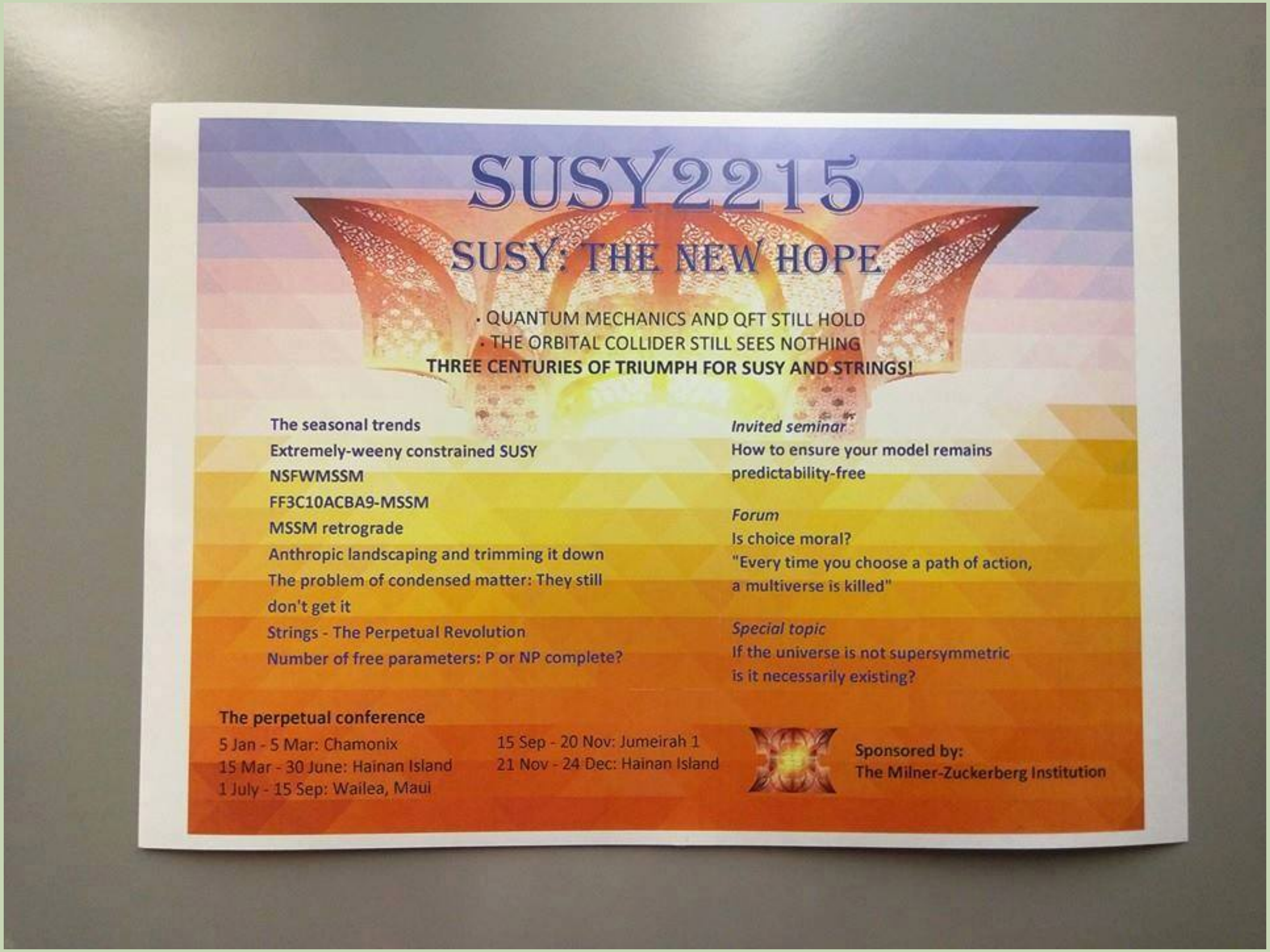
A very special symmetry that relates particles two basic classes of elementary particles – Bosons, which have an integer-valued spin, and Fermions, which have a half-integer spin.



Where is supersymmetry?

No one knows! It might be a figment of theoretical mind. We don't have any experimental evidence! Experimentalists have spent decades looking for it at the LHC. Condensed matter experimentalists have tried to search for the Tri-critical Ising (TCI) quantum criticality and the associated supersymmetry and haven't observed anything experimentally in spin models either (**arXiv:1602.08172**).

SUSY 2215! [Seen at CERN]



Another joke!

A famous theoretical physicist had an experimentalist friend and invited him for dinner. He told him -"I am pretty sure that my equations are correct and there must exist a new particle". The experimentalist collaborated with thousand other physicists, wrote grants and set up the experiment. They found nothing after one year. He went and told the theoretical physicist - "We found no new particle". Theoretical physicist stands up, shakes his head and says - "One full week of effort wasted".

Why supersymmetry? And on top of that lattice supersymmetry?

Motivation - Gauge/gravity (holography)

We do not yet understand the quantum theory of gravity but growing evidence that it has to be holographic in nature. First concrete example was proposed by Maldacena in 1997 which is now known as AdS/CFT correspondence.

General idea: 't Hooft and Susskind (1993,1994)
D-brane revolution: Polchinski (1995)

Gauge/gravity duality

- 4d $\mathcal{N} = 4$ U(N) SYM theory associated with the world-volume of stack of N D3-branes is dual to Type IIB supergravity (SUGRA) on anti-deSitter in five dimensions + five-sphere in the decoupling limit (large N, λ).

- Lower dimensions (non-conformal cases) – Maximally supersymmetric (p+1)-dimensional YM theory dual to Dp-branes at low temperatures - dual description in terms of Type IIA/B (even/odd p) low-energy string theory. We will not discuss anything about $p > 3$.

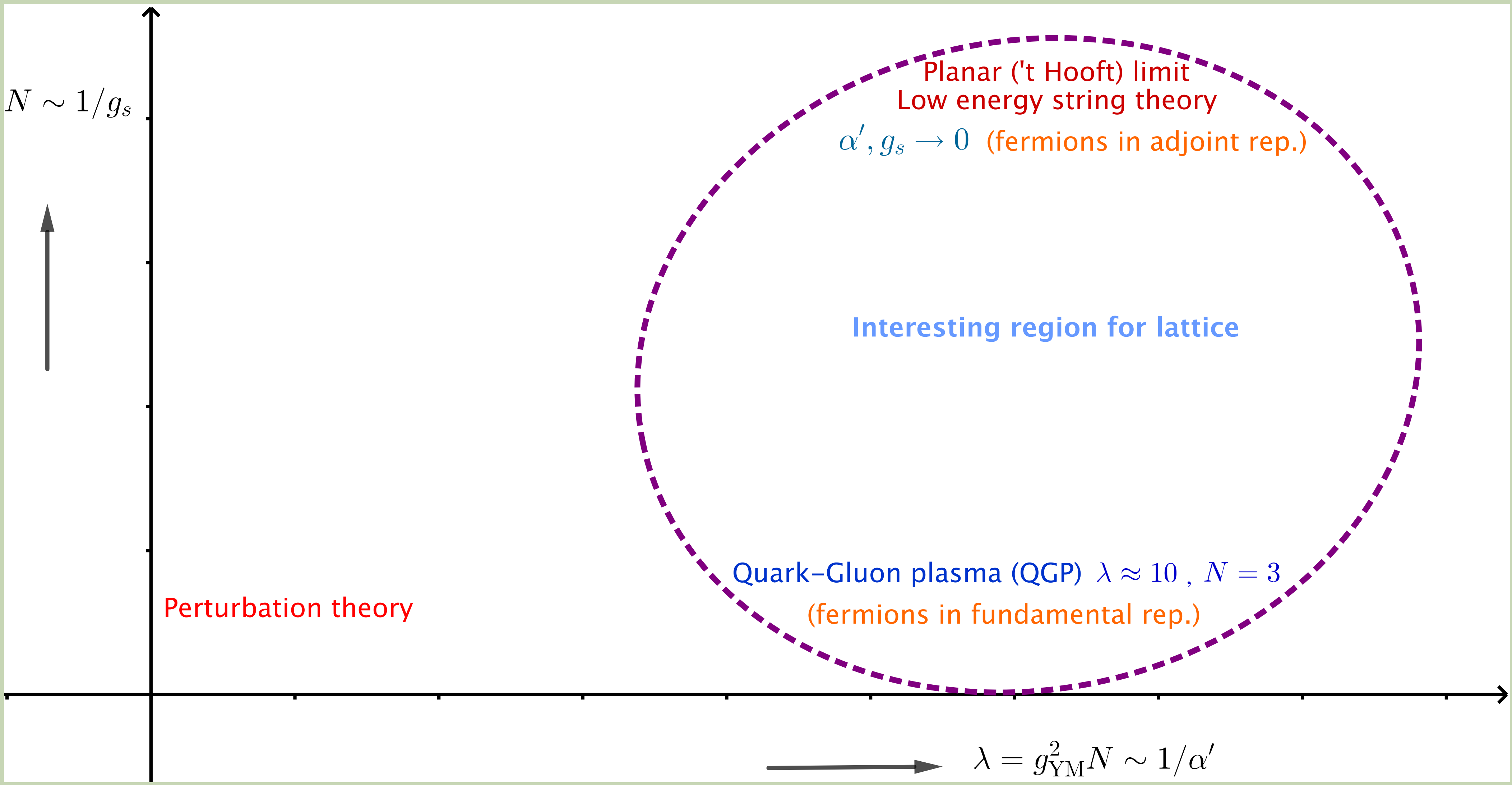
$\mathcal{N} = 4$ SYM theory

Action is schematically given by,

$$S = \frac{N}{\lambda}\int d^4x \quad \mathrm{Tr}\left\{\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(D_\mu X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + S_{\text{fermions}}\right\}$$

- SU(N) gauge theory with four fermions and six scalars, all massless and in adjoint rep.
- Supersymmetric: 16 supercharges
- Fields and Q's transform under global SO(6) R-symmetry
- Conformal: β function is zero for all 't Hooft coupling, $\lambda = g_{YM}^2 N$

(S)YM theory



$N \sim 1/g_s$
Planar ('t Hooft) limit
Low energy string theory
$\alpha', g_s \to 0$ (fermions in adjoint rep.)
Interesting region for lattice
Quark–Gluon plasma (QGP) $\lambda \approx 10$, $N = 3$
(fermions in fundamental rep.)
Perturbation theory
$\lambda = g_{\rm YM}^2 N \sim 1/\alpha'$

Lattice and $\mathcal{N} = 4$ SYM

- We know that to understand QCD at strong couplings, lattice has been very successful. So, let's apply it to SUSY theories!
- Several complications because of supersymmetric algebra, field content, conformal symmetry, holographic limits, and fermions!!
- Some progress in the last decade! Use ideas such as twisting of supersymmetric theory, Kaehler-Dirac fermions, A4* lattice.
- 4d theory at strong couplings still a distant dream!

Topological twisting

$$SO(4)_{\rm tw} \equiv {\rm diag}\left[SO(4)_{\rm Euc} \times SO(4)_R\right]; \quad SO(4)_R \subset SO(6)_R$$

- Q's transform with integer spin under this new rotation group.
- Results in one scalar supercharge (0-form), which is exactly preserved on the lattice. This construction similar to one by orbifolding.
- Not all supersymmetric (SUSY) theories possible to study on the lattice. In fact, $\mathcal{N}$=4 SYM is the only theory (in 4d) possible to study on lattice! No complains here!

Lattice $\mathcal{N} = 4$ SYM

- Q's transform with integer spin under this new twisted rotation group. Gives a scalar supercharge which is exactly preserved on the lattice.

- Discretise on a special A4* lattice using complex gauge links which involves six scalars and four gauge fields of the original theory. Fermions are split as $16 \rightarrow 1 \oplus 5 \oplus 10$

- The irreps of the point group symmetry of the lattice matches those of the SO(4)′ twisted rotation group.

- Gauge invariance, lattice symmetry, Q-exact supercharge is highly constraining and limits the number of counter terms needed to get to the continuum limit

$U(N) = U(1) \otimes SU(N)$

The exact $\mathcal{Q}$ acts on the gauge links as, $\mathcal{Q}\mathcal{U}_a = \psi_a$. The fermions have expansion in terms of generators. Similarly, the complex gauge fields are represented as Wilson gauge fields which take their values in the algebra of U(N) gauge group.

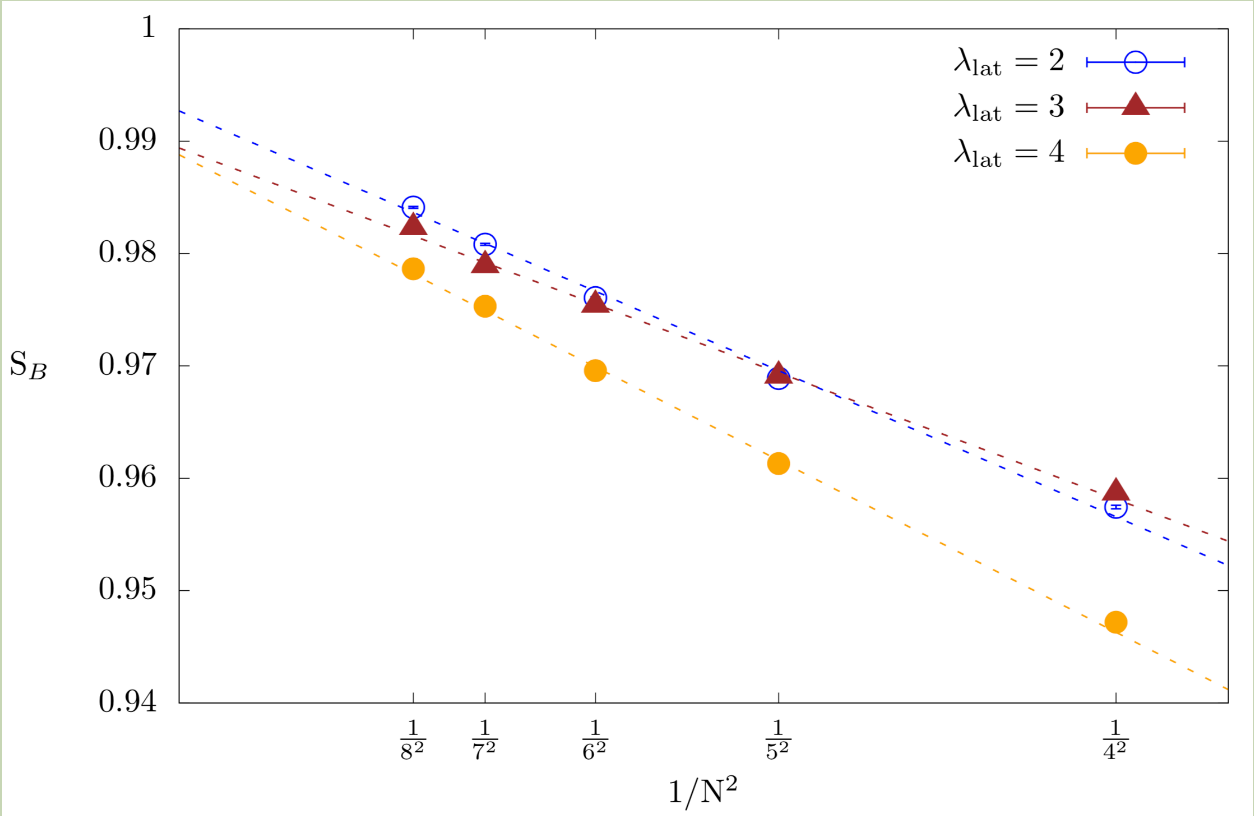
To obtain the correct naive continuum limit, the complexified gauge links must have the expansion $\mathcal{U}_a = \mathbb{I} + a\mathcal{A}_a + \cdots$
in some appropriate gauge.

U(1) trace mode [Catterall, Giedt, RGJ - arXiv 1808.04735]

It is known that this U(1) which is irrelevant in the continuum theory creates problem for the lattice theory. We explored how adding a term to the lattice action which removes this U(1) mode affects the Q-exact quantities which is a feature of the SU(N) theory.

The deviation goes down as $\sim 1/N^2$ and in the holographic limit reproduces the same holographic predictions as the original theory.

Results in the large N limit



$S_B = 1 \quad \rightarrow \mathcal{Q}-\text{exact}$, so this truncation is a reasonable way to deal with lattice instabilities. Still not good for strong couplings!

Dimensionally reduced SYMs

- Reduce the four dimensional theory down to one dimension (time) to get BFSS (Banks-Fischler-Shenker-Susskind) model.
- Add mass deformation to this theory to study matrix model on pp-wave space-time, known as PWMM or BMN (Berenstein-Maldacena-Nastase) model
- Reduce the theory to (1+1)- dimensions to study SYM theory dual to Type IIB supergravity (SUGRA) having different black hole solutions (*uniform* black string and *localised* black hole) and transition between them.

Supergravity regime (p<3)

To have a valid SUGRA limit, we need:

- Radius of curvature large in units of α', which means $\lambda \gg 1$
- The string coupling, $g_s \ll 1$, which means that *N* should be large.

These two combined gives the following : $1 \ll \lambda_p \beta^{3-p} \ll N^{\frac{10-2p}{7-p}}$,

Note that for *p*=3, this reduces to the familiar limit.

BFSS matrix model

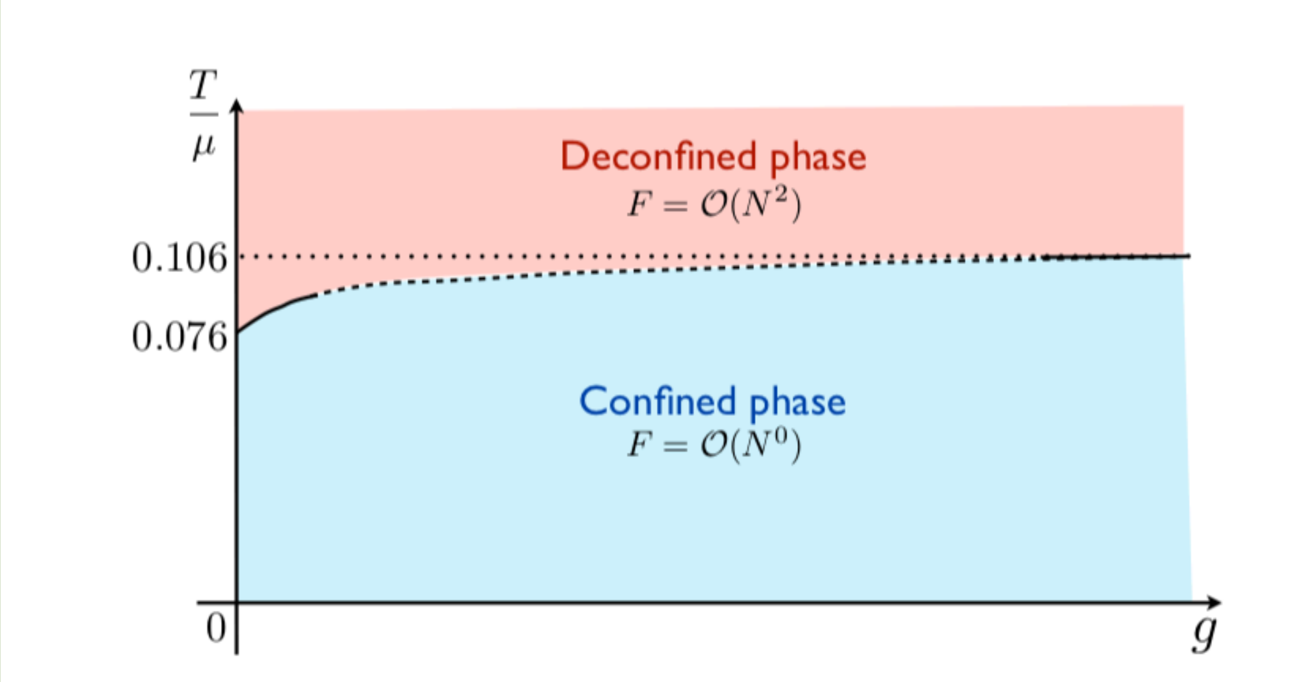
$$S_{\mathrm{BFSS}} = \frac{N}{4\lambda}\int dt \mathrm{Tr}\left[(D_t X^i)^2 - \frac{1}{2}\left[X^I, X^J\right]^2 + \Psi^T D_t \Psi + i\Psi^T \gamma^j [\Psi, X^j]\right]$$

This model can be obtained by the dimensional reduction of $\mathcal{N} = 1$ SUSY in ten dimensions.

I, J runs from 1 … 9, and we have total of sixteen Ψ , where all fields are N x N matrices and in the adjoint representation of the gauge group SU(N).

Matrix model with phase structure

Can we find study some holographic model with an interesting phase structure? BFSS matrix model only has a *deconfined* phase. However, when we consider the massive deformation of this model (known as BMN or PWMM) model on a non-flat space-time, there is an interesting phase structure.



Different black hole solutions

There are black-hole uniqueness theorems which show that the topology of black holes in 4d space-time are spheres. In higher dimensions, this this changes and we can have cylindrical and spherical black holes depending on the solution that maximizes entropy.

Clearly, these transitions are very interesting in view of holography because they are known to be dual to deconfinement transitions. In general, lot of physics can be understood via the phase structure.

Lattice study of BMN model

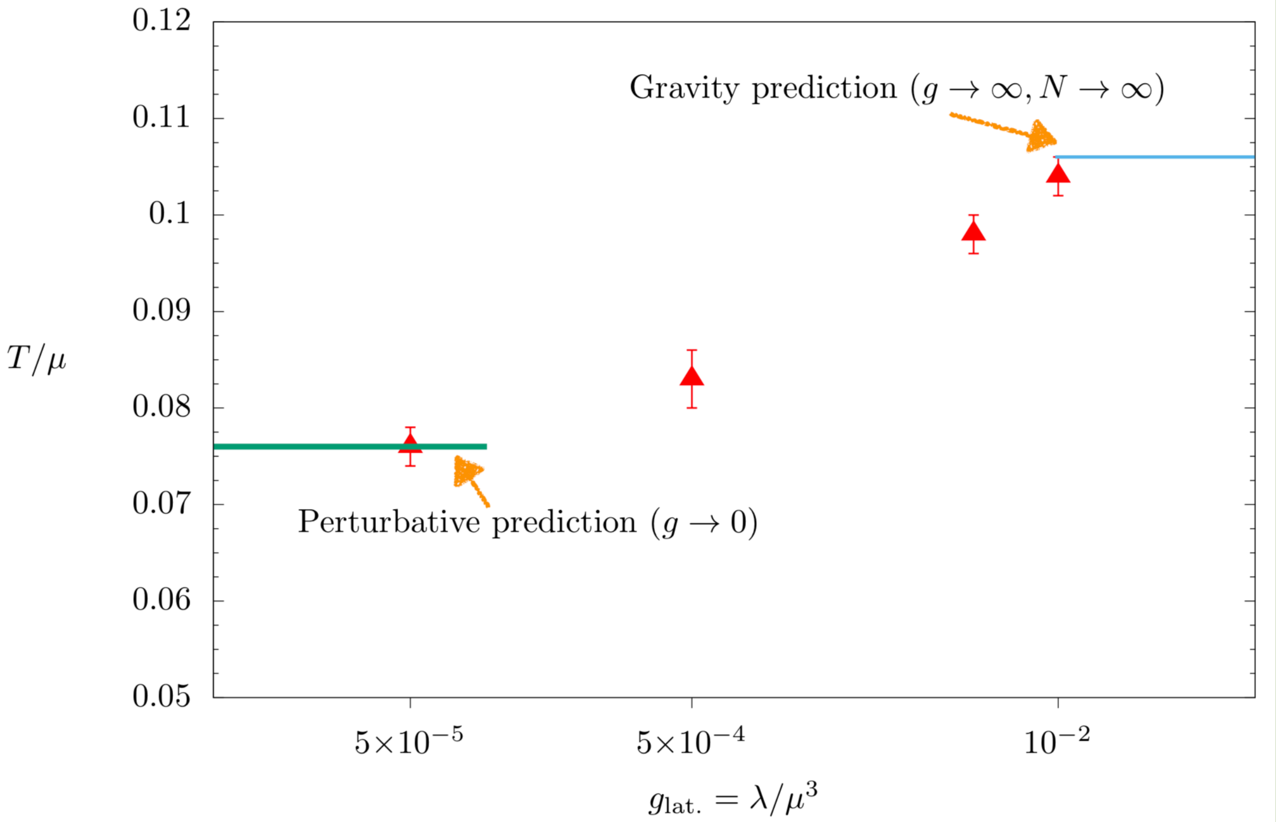
[Catterall, RGJ, Joseph, Schaich, Wiseman arXiv 19XX.XXXXX]

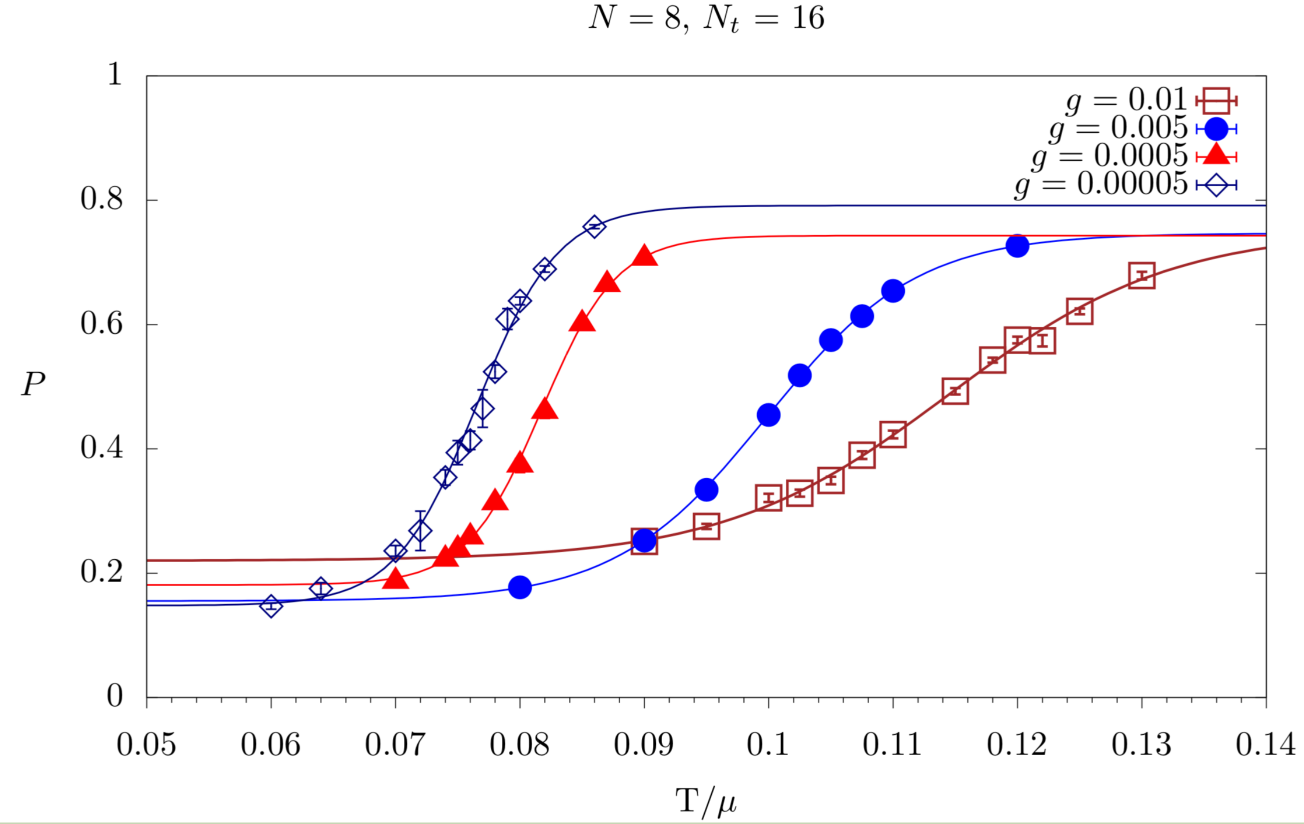
$$S_{BMN} = S_{BFSS} - \frac{N}{4\lambda}\int d\tau\ \mathrm{Tr}\left(\frac{\mu^2}{3^2}\left(X^I\right)^2 + \frac{\mu^2}{6^2}\left(X^i\right)^2 + \frac{2\mu}{3}\epsilon_{IJK}X^I X^J X^K + \frac{\mu}{4}\Psi^T\left(\gamma^{123}\right)\Psi\right)$$

The flat directions of the BFSS model are lifted by giving masses to SO(3) and SO(6) scalars. In addition, there is a cubic scalar term which is known as 'Myers term' plus a fermion term. Unlike BFSS, this model is on pp-wave spacetime. Even after the addition of these mass terms, supersymmetry is intact.

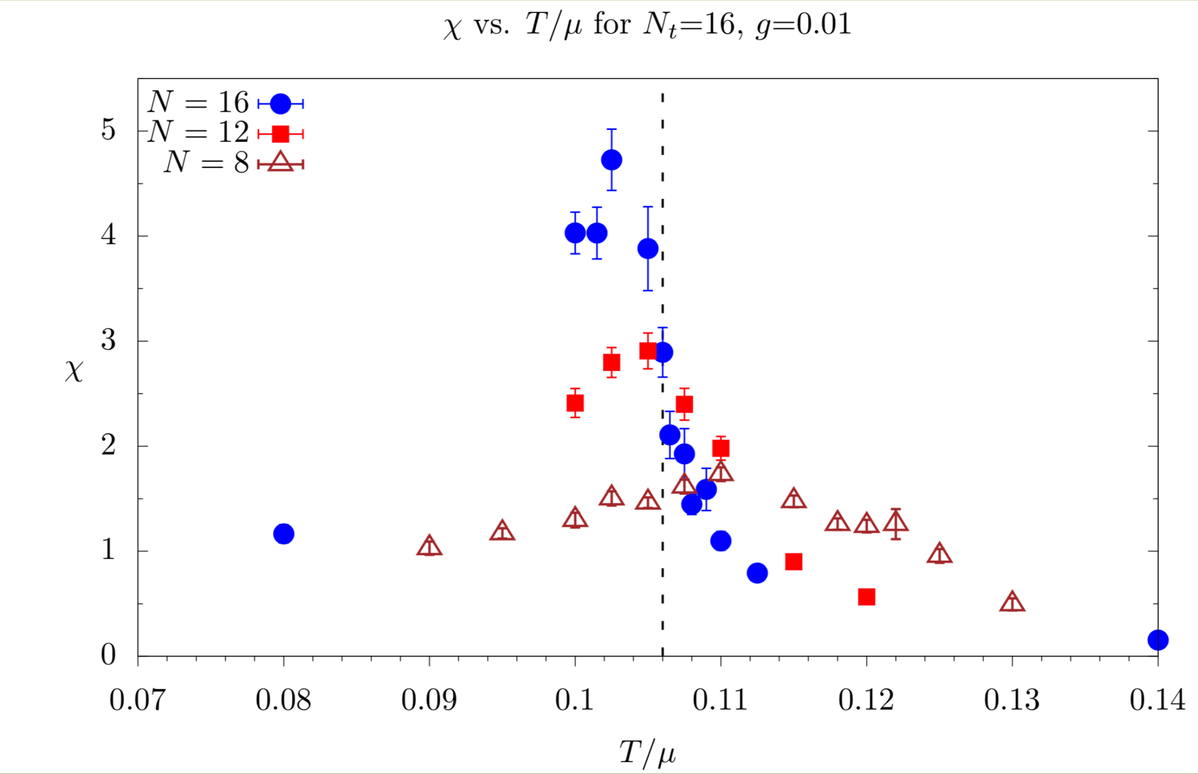
Dual classical gravity solution valid when N is large and coupling is strong.

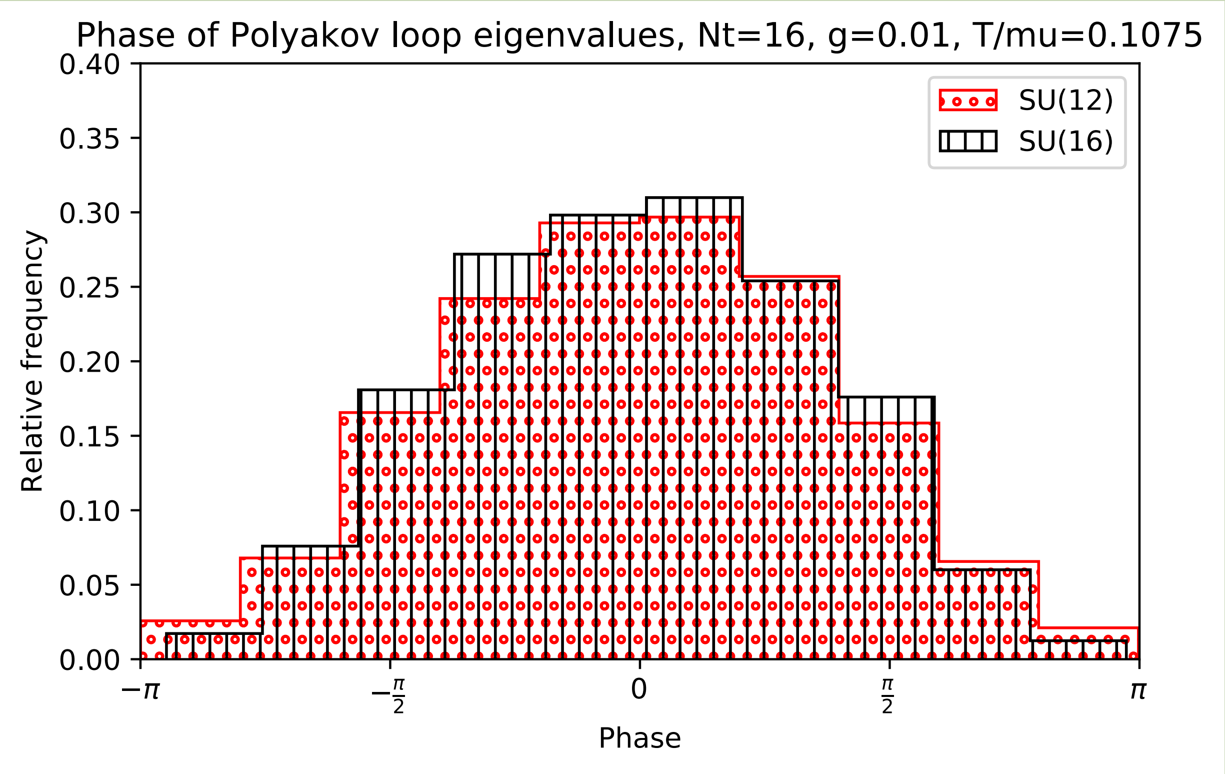
BMN model - Results (in progress!)





BMN model - Results





1+1-SYM

Reduce the 4d theory down to two dimensions. Only some region of parameter space in SYM theory has valid supergravity description which given by $1 \ll \cdots \ll N^{2/3}$

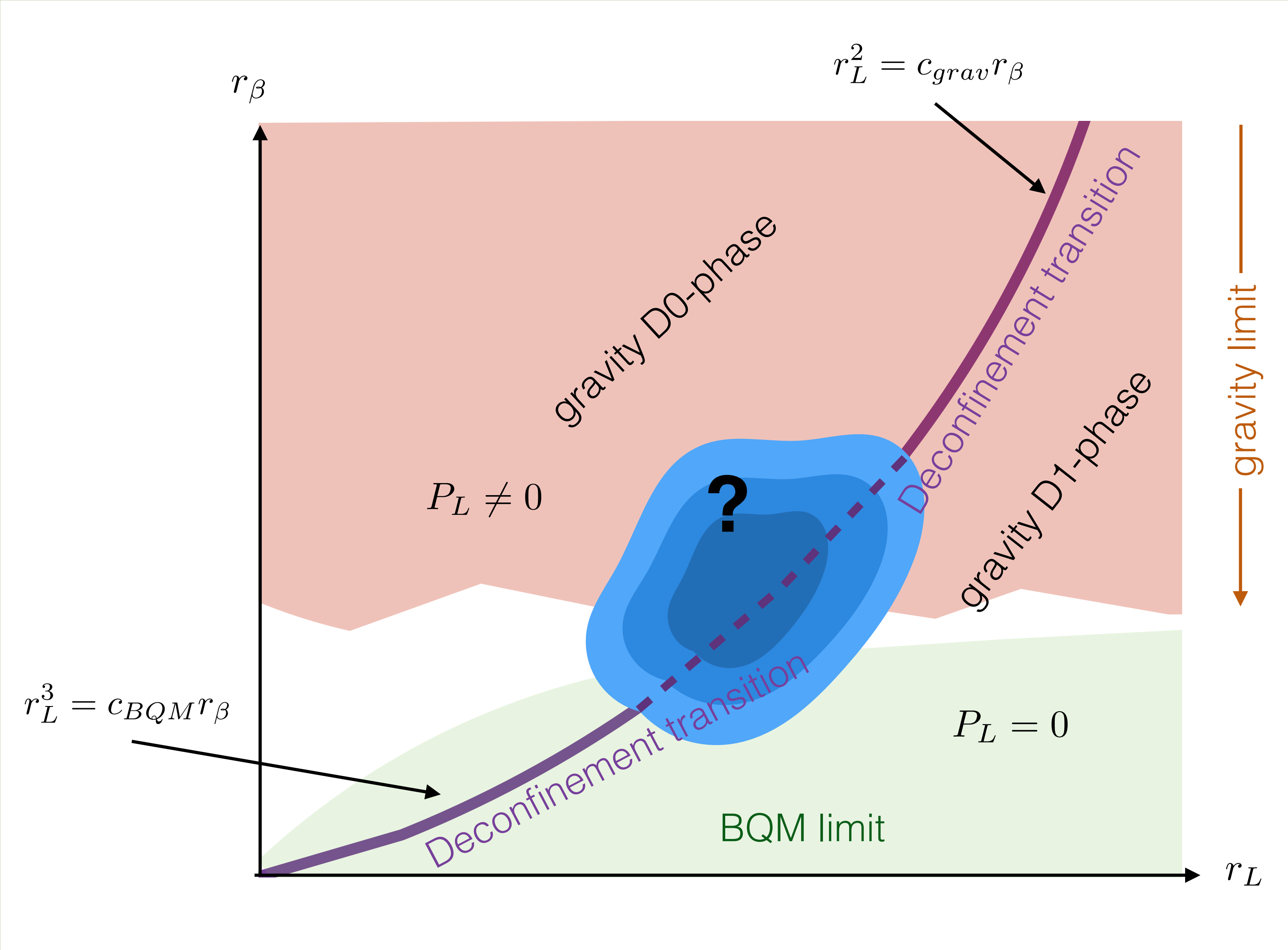
We measure Wilson line around the two cycles. The gravity solutions are static black holes, their Euclidean time circle is contractible so we expect a deconfined Polyakov line, $P_\beta \neq 0.$ The homogeneity of the horizon is taken to indicate that the eigenvalues of Wilson line are uniformly distributed at large N (for which we will see the results later).

1+1-SYM

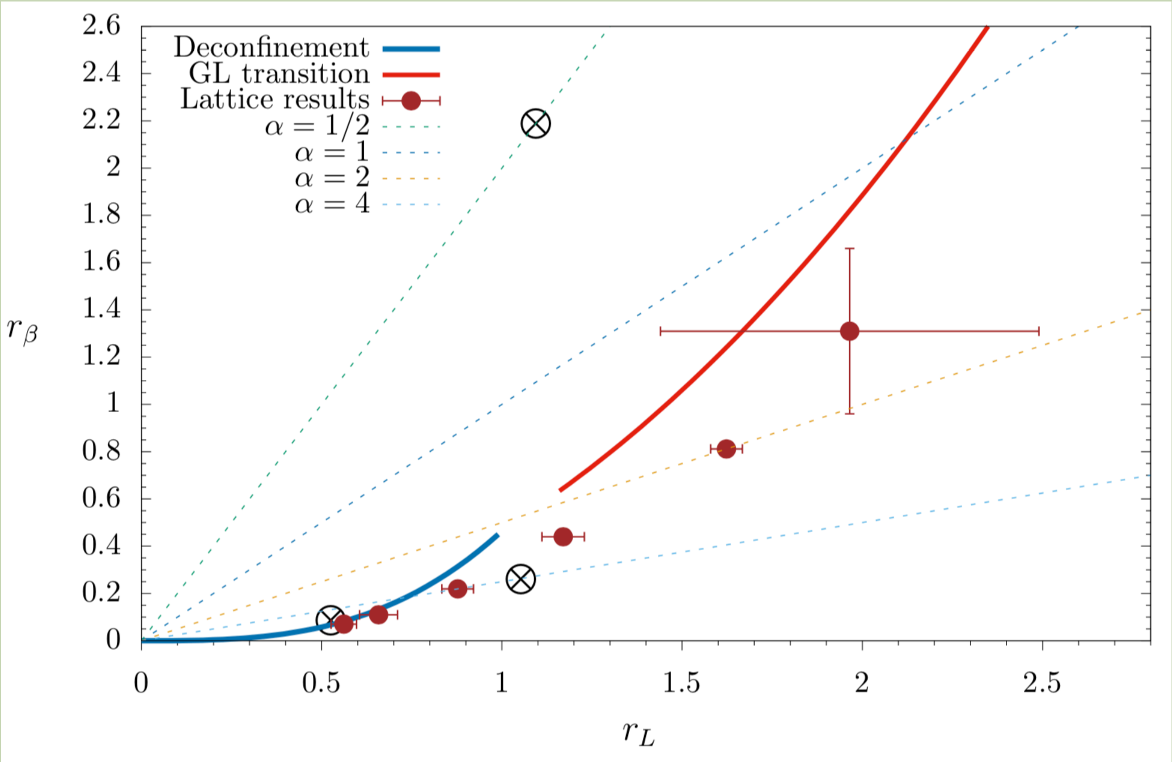
- Dimensionless coupling, $r_x = \sqrt{\lambda}L$, $r_\beta = \sqrt{\lambda}\beta$, $\alpha = L/\beta = r_x/r_\beta$
- Santos et al. calculation from gravity side matches old work for square lattice. Gregory-Laflamme transition between localized black hole and homogeneous black string at $\alpha^2 r_\beta = 2.45$
- This is a topological transition on the gravity side, dual to *deconfinement* transition on the gauge theory side. In the large N limit and weak coupling, there is well-known Gross-Witten-Wadia (GWW) transition.
- We study this theory on a reduced A4* –> A2* (triangular lattice)

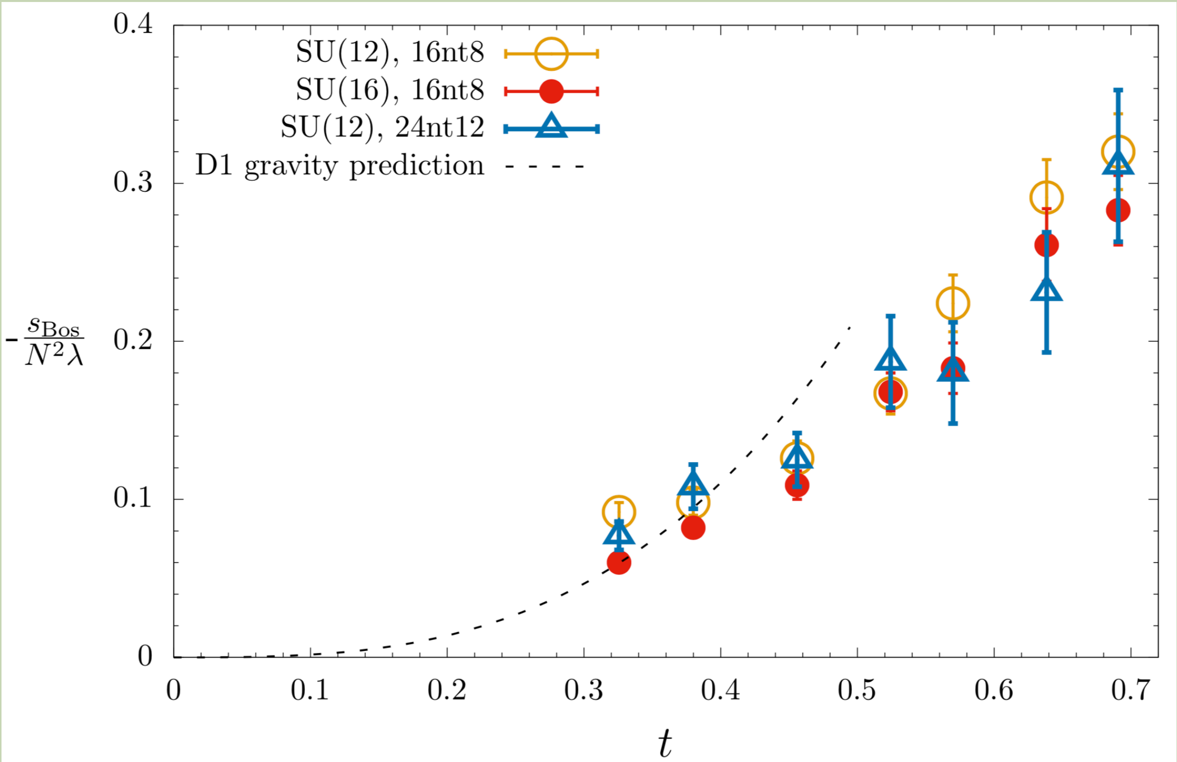
Phase diagram

At weak coupling (no gravity description, high-T), bosonic QM limit. As we enter gravity phase, we have different free energy behaviour for D0 & D1 phases. For D1 phase, we have $\frac{S_{Bos}}{N^2\lambda} = -1.73\ t^3$ and for D0 phase $\frac{S_{Bos}}{N^2\lambda} = -2.47\frac{t^{16/5}}{\alpha^{2/5}}$

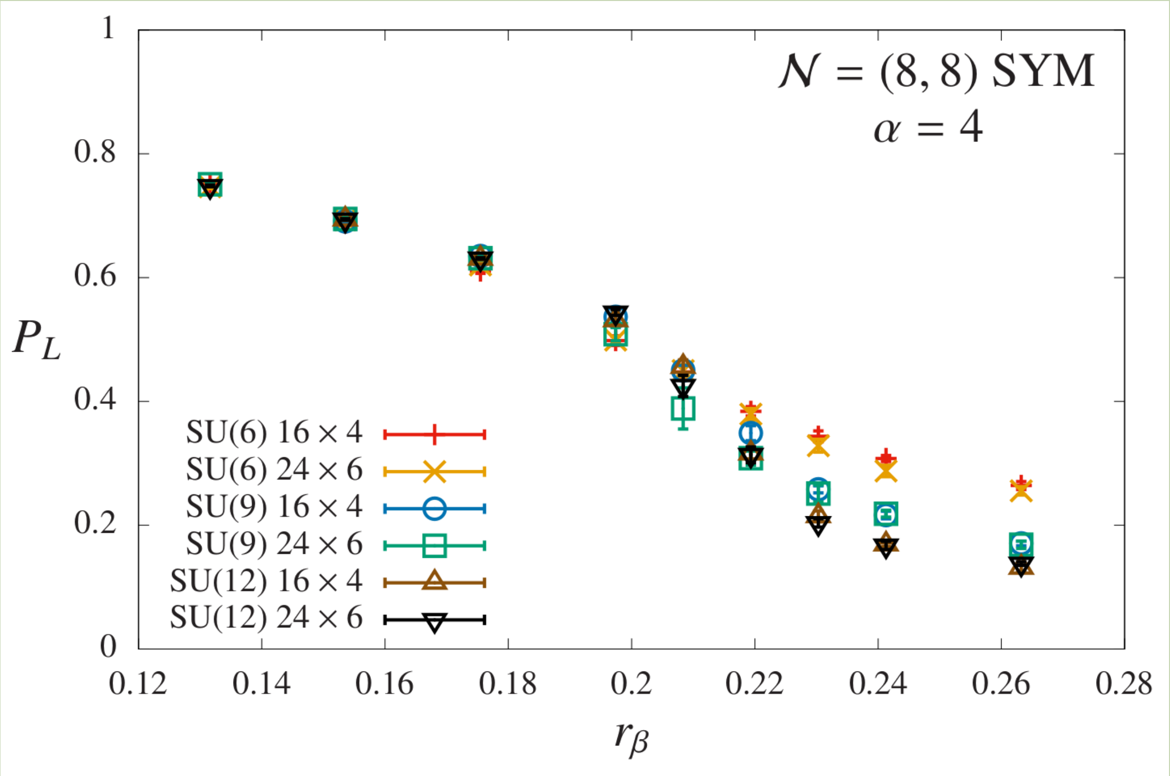


1+1-SYM [Catterall, RGJ, Schaich, Wiseman, arXiv 1709.07025]

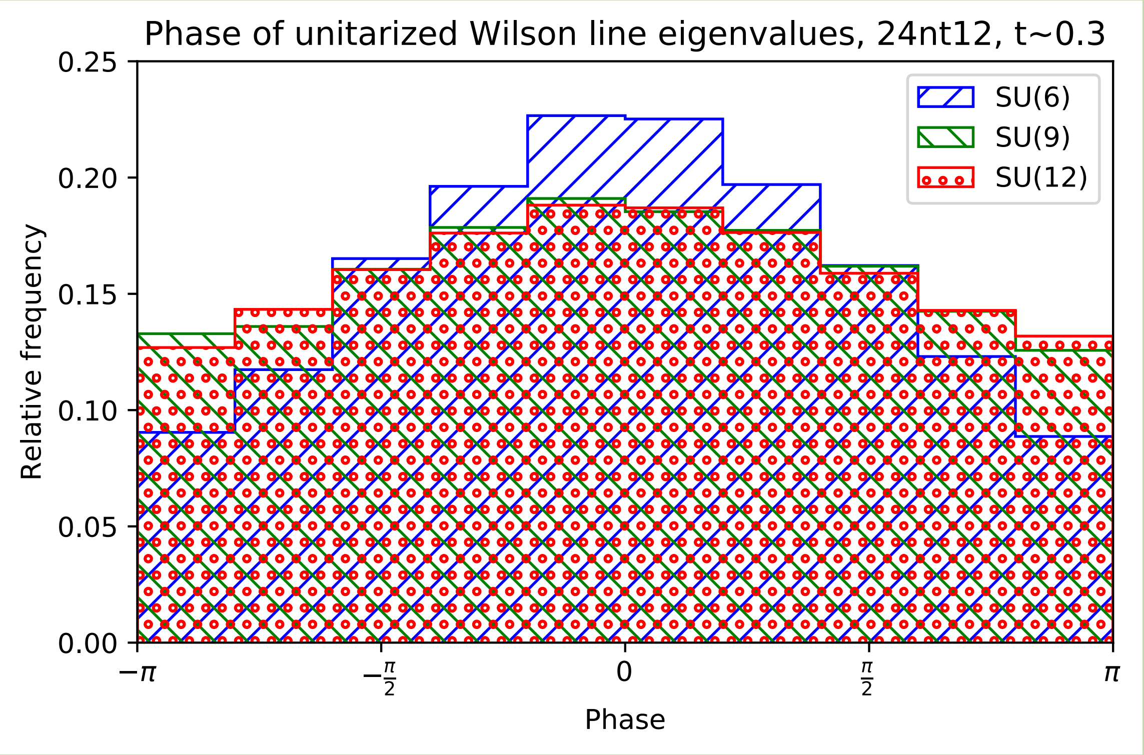




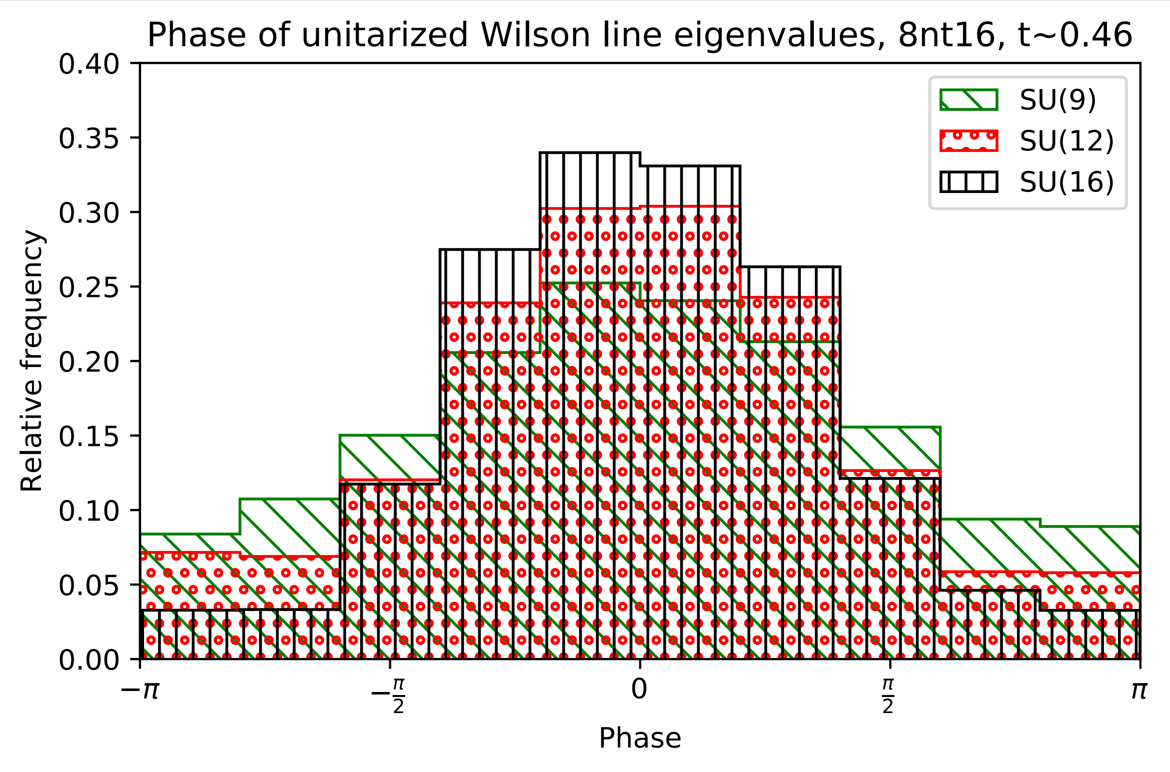
1+1-SYM



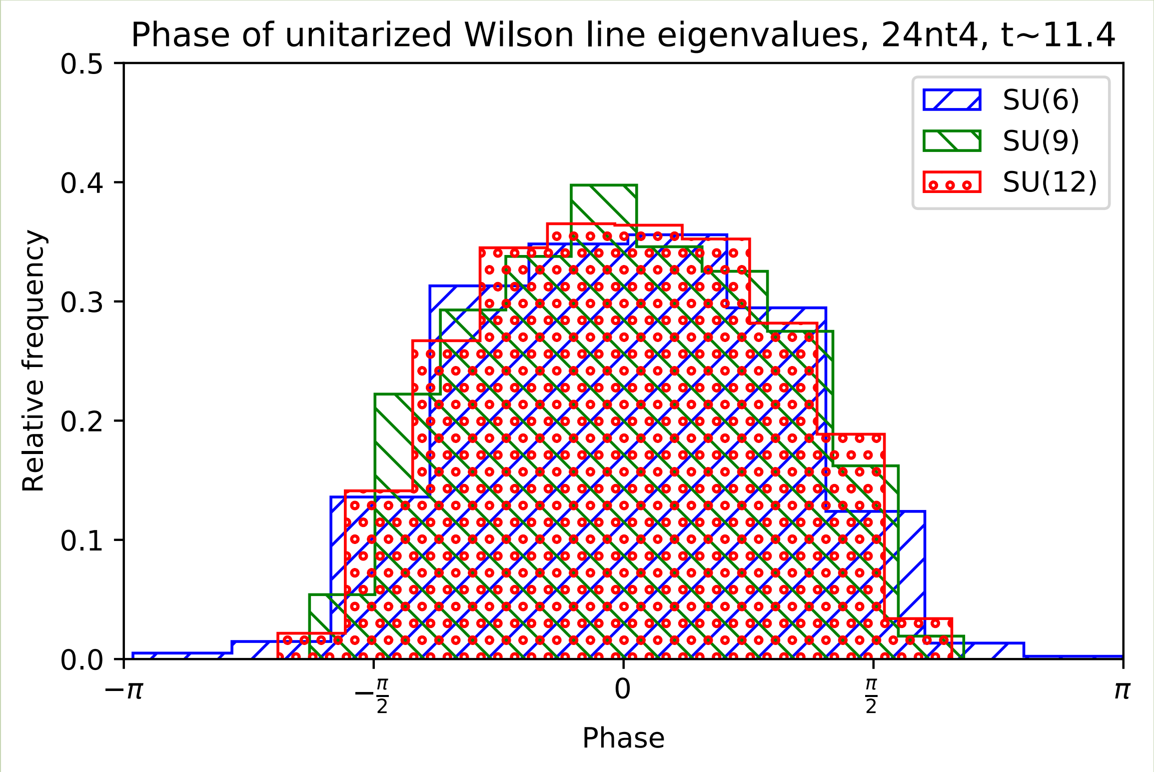
$\mathcal{N} = (8,8)$ SYM
$\alpha = 4$
P_L
r_β
SU(6) 16 × 4
SU(6) 24 × 6
SU(9) 16 × 4
SU(9) 24 × 6
SU(12) 16 × 4
SU(12) 24 × 6



Phase of unitarized Wilson line eigenvalues, 24nt12, t~0.3
Relative frequency
Phase
SU(6)
SU(9)
SU(12)



Phase of unitarized Wilson line eigenvalues, 8nt16, t~0.46
Relative frequency
Phase
SU(9)
SU(12)
SU(16)



Phase of unitarized Wilson line eigenvalues, 24nt4, t~11.4
Relative frequency
Phase
SU(6)
SU(9)
SU(12)

1+1-SYM (hydrodynamics) [RGJ, arXiv 1809.00797]

For $p \neq 3$, we have non-conformal branes and they have interesting hydrodynamical features. For example, the speed of sound in the strongly coupled field theory plasma has been argued to be

$$c_s^2 = \frac{(\partial P/\partial T)_V}{(\partial E/\partial T)_V} = \frac{S}{c_V} = \frac{5-p}{9-p}$$

It also signals the onset of a tachyonic instability that is because of the fact that for p > 5 the specific heat becomes negative and this implies that the speed of sound in such a plasma is imaginary. It gives $c_s^2 = 1/3$ for four-dimensional SYM which is conformal.

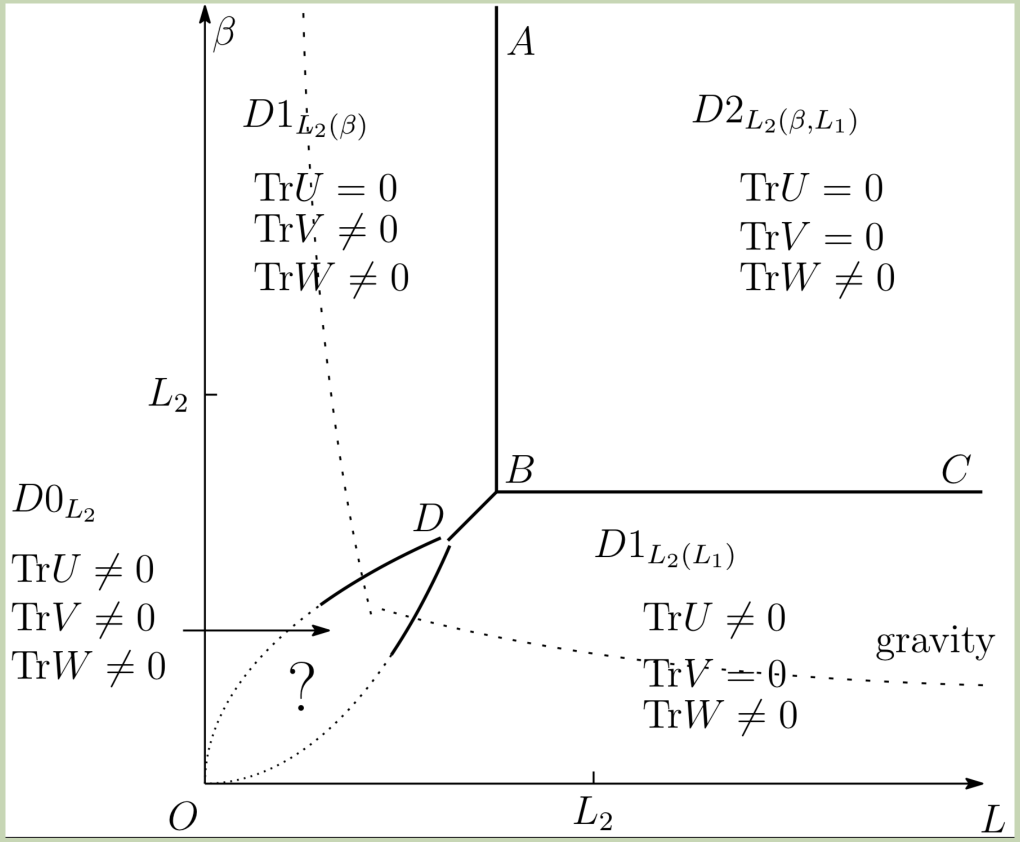
1+1-SYM (hydrodynamics)

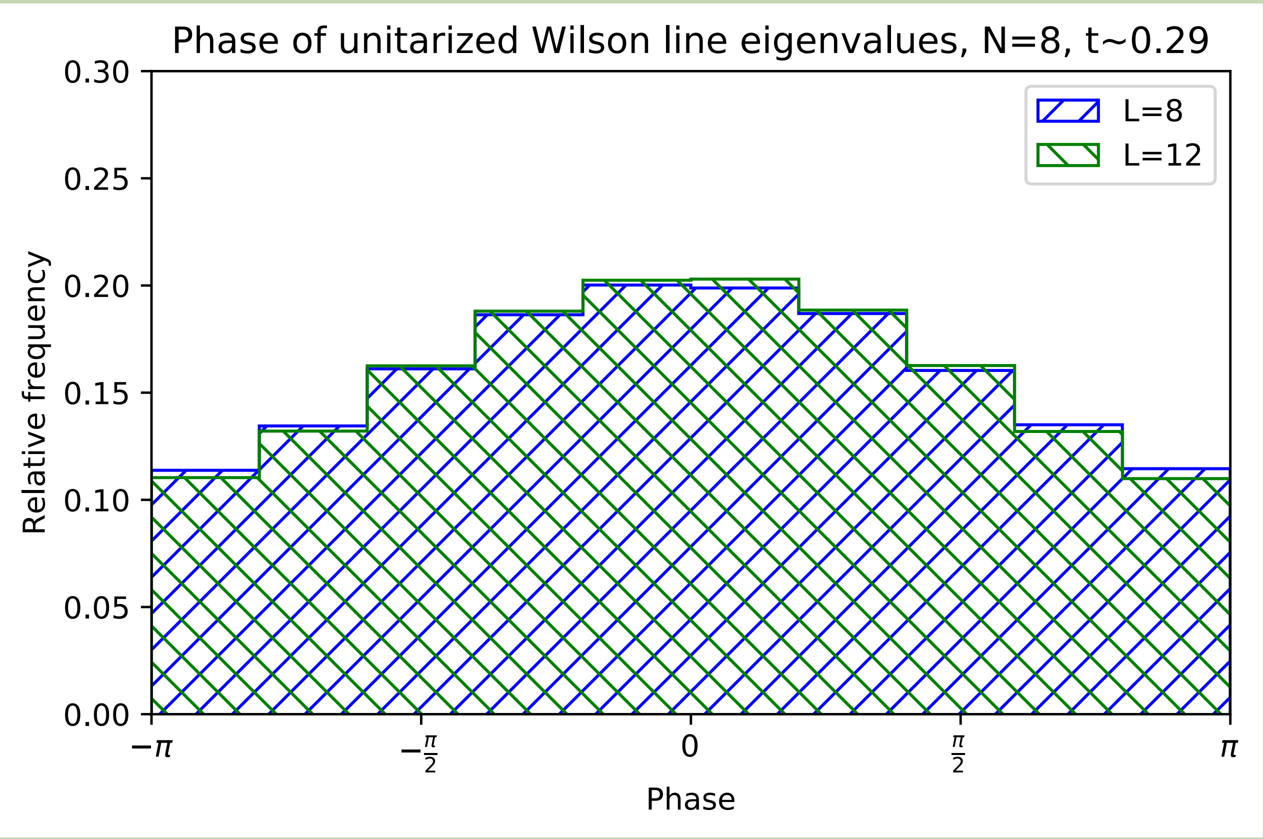
For p=1, it is known that $c_s^2 = 1/2$, but 1+1-SYM is not conformal so this result is bit unexpected. It does flow to CFT in the IR, but the result is from an intermediate regime where D1- description is valid.

It would be interesting to test this by calculating energy and pressure using Monte Carlo simulations. Determination of the equation of state (EoS) and speed of sound in QCD plasma is a well-pursued problem in lattice QCD. Plan to try it out for supersymmetric theories [holographic duals] !

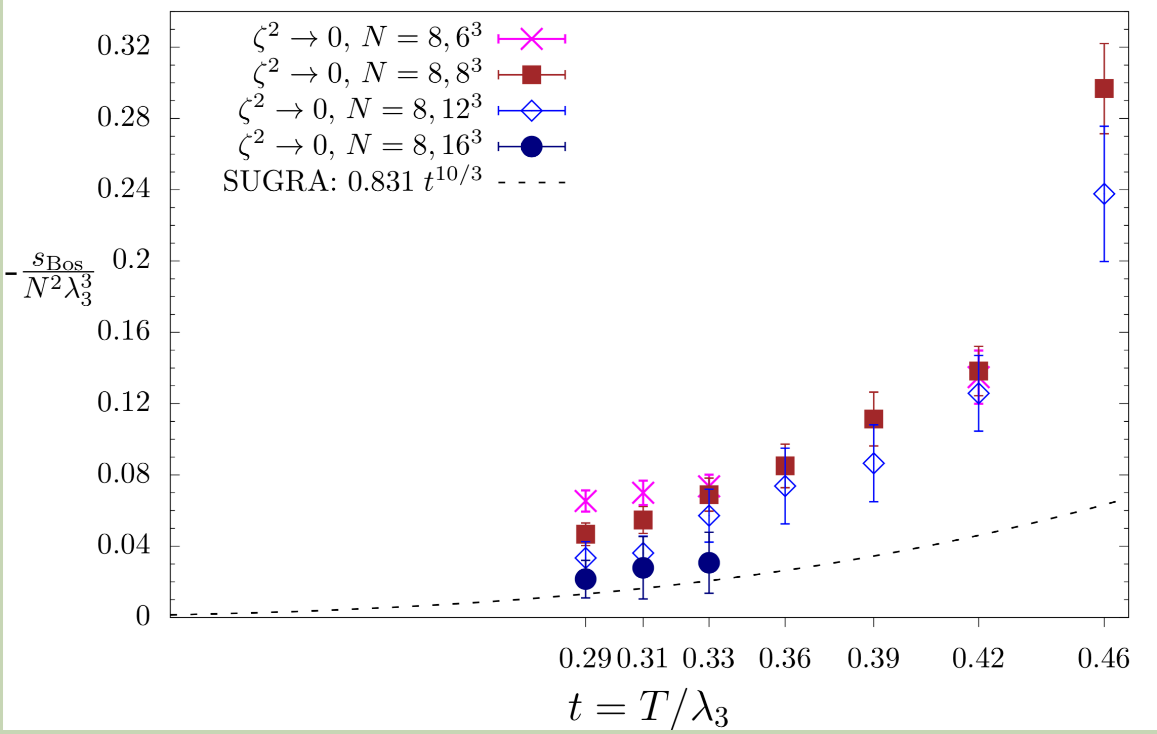
2+1-SYM [Catterall, Giedt, RGJ, Schaich, Wiseman arXiv 19XX.XXXXX]

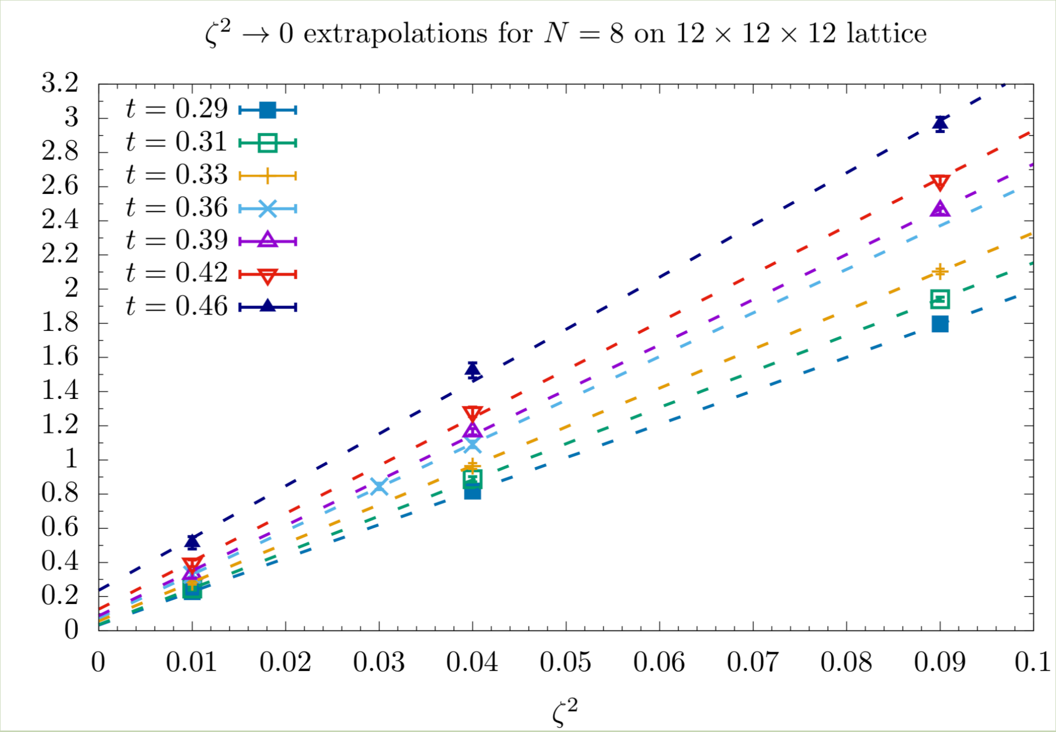
In a way similar to the 1+1 dimensional, we can understand the thermodynamics of D2-branes from 3d SYM theory with sixteen supercharges. The phase structure is complicated with D0, D1, and D2 phases. If the extents are same, and sufficiently large, we are in D2 phase where only the thermal cycle is *deconfined* i.e. $\mathrm{Tr}W \neq 0$

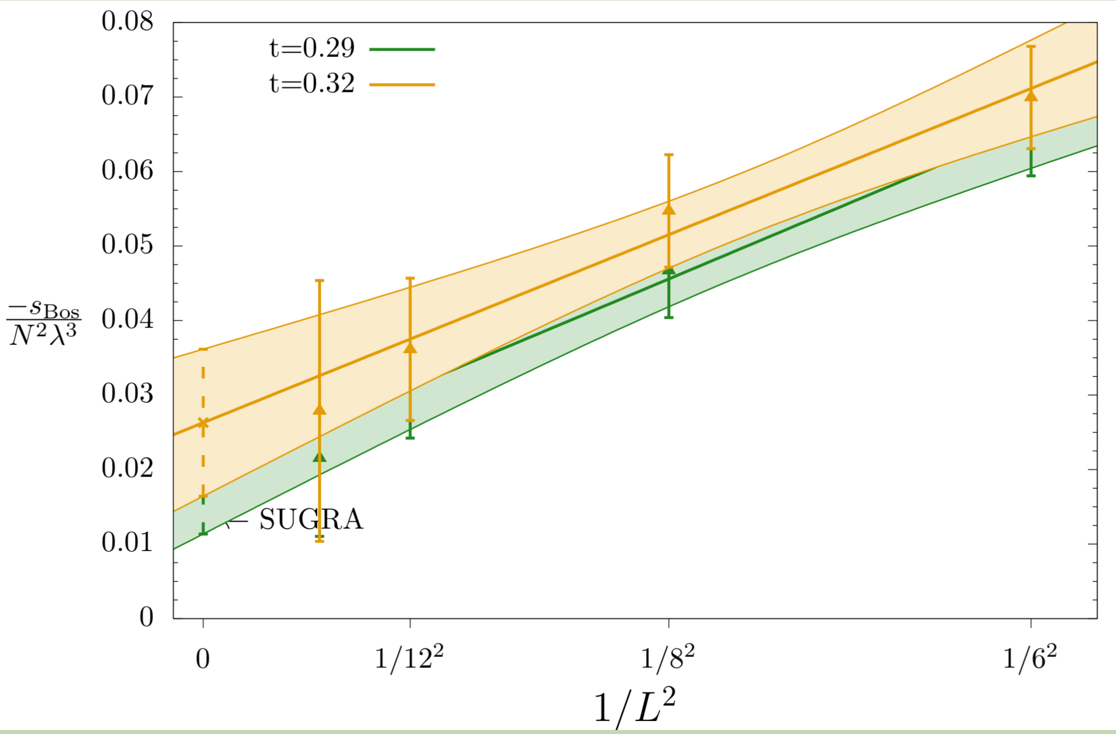




2+1-SYM [Preliminary]







Future directions

- Calculate other interesting quantities in lower-dimensional SYM and compare to holographic predictions like static-potential in 3d, Maldacena-Wilson loop in BFSS/BMN model, distribution of scalars in gauge theory.
- Explore entanglement entropy and possibility of alternative order parameter for *deconfinement* transition.
- Tensor networks and its relation to holography and bulk reconstruction.
- Develop parallel code over number of colors (N) to access planar limit in lower dimensional theories (especially BFSS/BMN).

Conclusions

We have made progress in studying strongly coupled supersymmetric field theories at strong couplings in $d < 4$ and compared qualitatively to the results expected from supergravity via gauge/gravity duality.

The four-dimensional theory is still difficult to study and we probably need some new and pathbreaking ideas.

We started to explore tensor networks for effectively studying gauge theories and plan to apply this numerical approach to systems which admit holographic dual in the next 3-4 years.

Exciting time for doing – "holographic" things!

Thank you!